Reply to "Comment on 'Abyssal Upwelling and Downwelling Driven by Near-Boundary Mixing'"

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ABSTRACT

Ledwell, in a comment on McDougall and Ferrari, discusses the dianeutral upwelling and downwelling that occurs near isolated topographic features, by performing a buoyancy budget analysis that integrates the diffusive buoyancy fluxes only out to a set horizontal distance from the topography. The consequence of this choice of control volume is that the magnitude of the area-integrated diffusive buoyancy flux decreases to zero at the base of a topographic feature resulting in a net dianeutral upwelling of water. Based on this result, Ledwell argues that isolated topographic features are preferential locations for the upwelling of waters from the abyss. However the assumptions behind Ledwell's analysis may or may not be typical of abyssal mixing in the ocean. McDougall and Ferrari developed general expressions for the balance between area-integrated dianeutral advection and diffusion, and then illustrated these general expressions using the very simple assumption that the magnitude of the buoyancy flux per unit area at the top of the turbulent boundary layer was constant. In these pedagogical illustrations, McDougall and Ferrari concentrated on the region near the top (rather than near the base) of isolated topographic features, and they found net sinking of abyssal waters. Here we show that McDougall and Ferrari's conclusion that isolated topographic features cause dianeutral downwelling is in fact a result that applies for general geometries and for all forms of bottom-intensified mixing profiles at heights above the base of such topographic features.

Microstructure observations in the Brazil basin have shown that small-scale turbulent mixing in the abyssal ocean is bottom intensified, implying that the dianeutral velocity in the stratified interior ocean is directed downward. Polzin et al. (1997), St. Laurent et al. (2001), and de Lavergne et al. (2016) realized that the zero flux condition at the sloping sea floor implies that there must be dianeutral upwelling in a bottom boundary layer. Klocker and McDougall (2010) considered the buoyancy budget for a complete ocean basin and showed that net upwelling requires that the area integral of the magnitude of the dianeutral diffusive buoyancy flux must increase with height, but their study did not shed any light on the physical balance that drives the dianeutral upwelling in the bottom boundary layer. In this way, Klocker and McDougall (2010) derived the buoyancy budget (Walin method) for the net upwelling across isopycnals but did not consider the downwelling and upwelling separately. Ferrari et al. (2016) showed that in order to have both net upwelling of bottom water and a reasonable vertical buoyancy stratification, it is essential for the ocean to have sloping sidewalls. They emphasized that the importance of the sloping sidewalls did not act through the increasing of the total ocean area with height but rather through the increase with height of the area of the ocean that is only a small distance above the bottom; this is the area of significant small-scale turbulent mixing.

McDougall and Ferrari (2017) performed a buoyancy budget analysis not only for the net upwelling integrated along an isopycnal but also for the sinking volume transport in the ocean interior [in the stratified mixing layer (SML)] as well as for the rising transport in the turbulent bottom boundary layer (BBL). The McDougall and

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Ferrari (2017) study amounted to the application of the Walin (1982) buoyancy budget method to the buoyancy surfaces that incrop at the sea floor, in contrast to the usual application of the Walin method to buoyancy surfaces that outcrop at the sea surface.

The key results of McDougall and Ferrari (2017) are that the dianeutral volume transports in the turbulent BBL $\varepsilon_{\mathrm{BBL}}$ and net diapycnal volume transport $\varepsilon_{\mathrm{net}}$ being the sum of $\varepsilon_{\mathrm{BBL}}$ and the diapycnal volume transport across the buoyancy surface in the SML $\varepsilon_{\mathrm{SML}}$ are given by the following (ignoring the geothermal heat flux at the sea floor):

$$\varepsilon_{\mathrm{BBL}} = \int \frac{\mathcal{B}_0}{b_z}\frac{1}{\tan\theta}\,dc, \quad \text{and} \tag{1}$$

$$\varepsilon_{\mathrm{net}} \equiv \varepsilon_{\mathrm{BBL}} + \varepsilon_{\mathrm{SML}} = \frac{dF}{db}. \tag{2}$$

The difference between these two equations gives an expression for $\varepsilon_{\mathrm{SML}}$. The line integral in Eq. (1) is performed along the entire perimeter where the buoyancy surface intersects topography. Here F is the magnitude of the diffusive buoyancy flux integrated across the whole interior area of an isopycnal:

$$F = \iint \mathcal{B}(b,x,y)\,dx\,dy, \tag{3}$$

where the area integral extends right into the BBL, although the contribution to F from the BBL is trivially small [see the discussion in McDougall and Ferrari (2017) before their Eq. (1) and after their Eq. (A5)]. The vertical profile of the magnitude of the diapycnal buoyancy flux $\mathcal{B}$ in the deep ocean is taken to be zero at the sea floor and to increase with height in the BBL to a maximum value of $\mathcal{B}_0$ at the top of the BBL of thickness h, and then to decrease exponentially with height (with scale height d) in the SML. Equations (1) and (2) are both the result of a Walin buoyancy budget for different control volumes, in the case of Eq. (1), for the BBL volume, and in the case of Eq. (2), for the entire volume between a pair of closely spaced buoyancy surfaces.

Assuming that the e-folding depth scale d is constant throughout the ocean, McDougall and Ferrari (2017) derived a simple expression for $\varepsilon_{\mathrm{SML}}$:

$$\varepsilon_{\mathrm{SML}} \approx -\frac{F}{\langle b_z\rangle d}, \tag{4}$$

where $\langle b_z\rangle$ is the average value of the vertical buoyancy gradient along the buoyancy surface.

By further integrating Eq. (2) with respect to buoyancy and substituting the resulting expression for F into Eq. (4), they arrived at the following diagnostic relationship between $\varepsilon_{\mathrm{BBL}}$ and $\varepsilon_{\mathrm{net}}$:

$$\varepsilon_{\mathrm{BBL}} \approx \varepsilon_{\mathrm{net}} + \frac{1}{\langle b_z\rangle d}\int_{b_{\mathrm{min}}}^{b} \varepsilon_{\mathrm{net}}\,db'. \tag{5}$$

From this relationship McDougall and Ferrari (2017) showed that while $\varepsilon_{\mathrm{BBL}} = \varepsilon_{\mathrm{net}}$ for the very densest water in the abyss, further aloft the upward dianeutral transport in the BBL $\varepsilon_{\mathrm{BBL}}$ is often several times as large as the net dianeutral transport $\varepsilon_{\mathrm{net}}$. We call this relationship, Eq. (5), a diagnostic one because it does not contain any explicit term in the diffusive buoyancy fluxes, but rather it is a diagnostic (or geometrical) relationship between two different dianeutral transports, $\varepsilon_{\mathrm{BBL}}$ and $\varepsilon_{\mathrm{net}}$.

McDougall and Ferrari (2017) estimated a diapycnal diffusivity $D_0 \approx 5\times10^{-3}\,\mathrm{m^2\,s^{-1}}$ was needed just above the BBL in order to achieve a BBL dianeutral transport $\varepsilon_{\mathrm{BBL}}$ of 100 Sv ($1\,\mathrm{Sv} \equiv 10^6\,\mathrm{m^3\,s^{-1}}$) in the abyssal ocean. This figure was based on the assumed perimeter of the abyssal ocean to be 5×10^7 m and the average value of $1/\tan\theta$ to be 400. Holmes et al. (2018) have provided more accurate values of the perimeter (viz., 4×10^8 m) and of the reciprocal of the slope in the abyssal ocean (viz., 240) (see their Fig. 11), so that using Eq. (1) we now estimate that a diapycnal diffusivity $D_0 \approx 10^{-3}\,\mathrm{m^2\,s^{-1}}$ is needed just above the BBL in order to achieve a BBL dianeutral transport of 100 Sv in the global abyssal ocean. These numbers must be taken with a grain of salt, given the radical approximations taken in estimating the integrals in Eq. (1) and (2).

Following early work by McDougall (1989), McDougall and Ferrari (2017) illustrated the use of the above relationships in idealized geometries, in all of which the maximum magnitude of the diffusive buoyancy flux per unit area $\mathcal{B}_0$ was taken to be a constant. One special case was a conical seamount, which exhibits a negative value of $\varepsilon_{\mathrm{net}}$ at all heights. An attempt was made in the discussion of their Fig. 6b to consider a more realistically shaped seamount, but the discussion was not exhaustive and did not consider what happens at the bottom of the seamount.

In section 10 of McDougall and Ferrari (2017) we showed that since F scales as $F \sim \mathcal{B}_0 Ld/\tan\theta$, with L being the perimeter of the buoyancy surface and $\tan\theta$ being the tangent of the angle the buoyancy surface makes with the sea floor, it follows that $\varepsilon_{\mathrm{net}}/F$ scales as

$$\frac{\varepsilon_{\mathrm{net}}}{F} = \frac{1}{F}\frac{dF}{db} \sim \frac{(\mathcal{B}_0)_b}{\mathcal{B}_0} + \frac{L_b}{L} - \frac{(\tan\theta)_b}{\tan\theta} + \frac{d_b}{d}. \tag{6}$$

The discussion of the seamount case in McDougall and Ferrari (2017) concentrated on the variation of the perimeter L with buoyancy, which is always negative, and we made a few preliminary remarks about the influence of the seamount slope (i.e. $\tan\theta$) on the right-hand side of Eq. (6). McDougall (1989) considered more seamount geometries than just the conical one, but in each case the effect of the seamount slope was ignored; that is, McDougall (1989) assumed that the horizontal width of the SML surrounding a seamount was constant, independent of the seamount's slope. This assumption seemed acceptable in 1989 but is inconsistent with the modern paradigm of taking the mixing intensity to vary exponentially with height above the BBL with a fixed e-folding height. The impact of variations in $\mathcal{B}_0$ and d were not considered in McDougall and Ferrari (2017) because there is too little data and theory to make general statements about their dependence on buoyancy along seamounts. The note of Ledwell (2018) instead focuses on such variations, as we show below.

Before addressing Ledwell's (2018) argument, we place the idea that isolated topographic features cause dianeutral sinking on a firm footing using two different theoretical lines of argument. The first argument is based on integrating Eq. (2) from a general buoyancy surface b_1 that intersects the topography at midheight, across which the magnitude of the area-integrated diffusive buoyancy flux is F_1, up to a buoyancy surface b_u that is several e-folding heights above the peak of the topography, across which the diffusive buoyancy flux is negligible. This integration gives

$$\int_{b_1}^{b_u} \varepsilon_{\text{net}}\, db' = -F_1 < 0, \tag{7}$$

so that if one chooses many evenly spaced buoyancy surfaces between these two levels, the average value of ε_{net} across these surfaces is negative. Hence, on averaging over a series of buoyancy surfaces, we find that isolated topography acts as a sink of the surrounding seawater above a given buoyancy surface. This is not to say that the dianeutral flow sinks along the seamount everywhere. Waters can well rise along the seamount in some regions, but sinking dominates on average.

The second argument has been made by Holmes et al. (2018) and is based on the buoyancy budget for the same control volume:

$$\int_{z(b_1)}^{z(b_u)} Q_{\text{in}}\, b\, dz' \;+\; b_1 \varepsilon_{\text{net}}(b_1) = F_1 > 0\,, \tag{8}$$

where Q_{in} is the rate of volumetric inflow per unit height into the control volume. When the continuity equation is multiplied by b_1 and subtracted from Eq. (8) we find

$$\int_{z(b_1)}^{z(b_u)} Q_{\text{in}}\,(b - b_1)\, dz' = F_1 > 0, \tag{9}$$

which implies that there must be some positive nonzero volume flux Q_{in} of surrounding seawater into the control volume. Returning to Eq. (8) and writing the vertical integral as the sum of increments over a vertical distance increment δz in terms of the volume transports $\varepsilon_i^{\text{in}} = Q_{\text{in},i}\delta z$ if $Q_{\text{in},i}$ is positive, and $\varepsilon_j^{\text{out}} = -Q_{\text{in},j}\delta z$ if $Q_{\text{in},j}$ is negative, this buoyancy budget Eq. (8) can be written as follows:

$$\sum_{i=1}^{I} b_i \varepsilon_i^{\text{in}} > \sum_{j=1}^{J} b_j \varepsilon_j^{\text{out}}. \tag{10}$$

Depending on the sign of $\varepsilon_{\text{net}}(b_1)$, $b_1\varepsilon_{\text{net}}(b_1)$ is included on either the left- or right-hand side of Eq. (10) as either $b_1\varepsilon_1^{\text{in}}$ or $b_1\varepsilon_1^{\text{out}}$. This version of the buoyancy budget shows that the transport-weighted average buoyancy of the water flowing into the control volume exceeds the transport-weighted average buoyancy of the water that exits the control volume. This result applies independently of the sign of $\varepsilon_{\text{net}}(b_1)$, and it shows that the upper region of an isolated topographic feature (whether a seamount or a ridge) acts as a sink of seawater from the surrounding ocean. This upper-layer water is transported dianeutrally to denser layers. That is, above their flanks, isolated topographic features cause a net sinking motion through buoyancy surfaces. This is a much more general statement than was stated or derived by McDougall and Ferrari (2017). Note that Eq. (10) provides a constraint on the net dianeutral sinking motion in the whole control volume for every choice of b_1, whereas Eq. (7) is a constraint on the average (over many different buoyancy surfaces) of the inflowing volume flux across the vertical walls of the control volume.

This brings us to the contribution of Ledwell (2018) where mixing in the vicinity of localized topography (both seamounts and ridges) is considered. The key point of difference between Ledwell (2018) and McDougall and Ferrari (2017) is that Ledwell (2018) conducts a buoyancy budget only out to a fixed radius from the seamount of his Fig. 1 and a fixed lateral distance from the ridge topography of his Fig. 2. The rationale is that the mixing, and hence $\mathcal{B}_0$, is enhanced above sloping topography through the generation and breaking of internal waves, while it drops rapidly over the abyssal plains around the seamount/ridge. At the deepest part of the seamount Ledwell's lateral restriction limits the area of the SML and so limits the magnitude of the area-integrated diffusive buoyancy flux F of these dense surfaces. Because of this assumption, F starts from zero at the base of the seamount of his Fig. 1 (and at the base of the ridge of his Fig. 2), with

F_z and hence $\varepsilon_{\mathrm{net}}$ being positive immediately above these depths. In both figures, the net dianeutral transport $\varepsilon_{\mathrm{net}}$ is negative at heights more than a few hundred meters above the base of the topographic feature.

Ledwell's idealization can be contrasted to the idealized case where the maximum magnitude of the buoyancy flux per unit area $\mathcal{B}_0$ on each vertical cast is constant and independent of the topographic slope. Under this idealized case, near the base of a realistic (nonconical) seamount the magnitude of the area-integrated diffusive buoyancy flux F increases strongly with depth (due to both the increasing circumference and the decreasing slope) so that $\varepsilon_{\mathrm{net}}$ is negative. The downward diffusive buoyancy flux across a near-bottom buoyancy surface causes the fluid below this surface to increase its buoyancy. In a steady state there must be lateral advection of dense (negative buoyancy anomaly) AABW toward the seamount. As a consequence, on buoyancy surfaces just above the sea floor there will be a net flow of water away from the seamount, carrying away the sum of 1) the downward directed transport $\varepsilon_{\mathrm{net}}$ that is caused by the strong vertical variation of F, and 2) the inflowing bottom transport of AABW.

Ledwell's (2018) curtailing of the buoyancy budget at a fixed distance (of 20 and 1000 km in Figs. 1 and 2, respectively) from the center of the topographic feature has just the opposite effect to what would occur if the full buoyancy budget was performed along the complete buoyancy surfaces at these depths because as the topographic slope weakens near the base of the topography, so the area of the SML increases. In Eq. (6) it is seen that this increasing slope with buoyancy tends to cause $\varepsilon_{\mathrm{net}}$ to be negative. Ledwell's assumption that there is no diffusive buoyancy flux outside a fixed radius is equivalent, in its effect on net upwelling, to assuming that $\mathcal{B}_0$ decreases rapidly toward the base of the topography and that this variation overwhelms the $-(\tan\theta)_b/\tan\theta$ term. That is, Ledwell has effectively ensured that near the base of the topography the term $(\mathcal{B}_0)_b/\mathcal{B}_0$ on the right-hand side of Eq. (6) is positive and sufficiently large so as to overcome the negative influence of $-(\tan\theta)_b/\tan\theta$ and, in the seamount case, also of L_b/L, which is also negative in this region. It is difficult to assess whether this result is generic, because it relies on detailed knowledge of the relative variations of parameters that are dynamically linked ($\mathcal{B}_0$, $\tan\theta$, d). McDougall and Ferrari (2017) avoided the issue by focusing on the effect of each parameter separately.

We note that Holmes et al. (2018) have performed a series of simple but more realistic examples than did McDougall and Ferrari (2017), and they have found that the nature of the net upwelling (or net downwelling) in the vicinity of a seamount depends sensitively on the shape of the topography, because changes in the topography affect the variation of both slope and circumference as a function of buoyancy. As an example, if the height of the seamount varies quadratically with radial distance, then once one is deeper than the very top region of the seamount, there is no net upwelling or downwelling. The corresponding behavior was already apparent in Fig. 7 of McDougall and Ferrari (2017) at the bottom of a bowl-shaped ocean whose height varied quadratically with radius. In that case there was no net upwelling even in the case with $\mathcal{B}_0$ being a constant.

In summary, while McDougall and Ferrari (2017) concentrated their remarks about isolated topographic features on the flow induced by bottom-intensified mixing in the region above the flanks of the topography, by contrast Ledwell (2018) has explored the consequences of a decrease in the magnitude of the buoyancy flux at the top of the BBL $\mathcal{B}_0$, close to the base of isolated topographic features. Turbulent mixing measurements from the Drake Passage do indeed show that the magnitude of near-bottom mixing is higher toward the top of oceanic ridges than toward their base (Mashayek et al. 2017). Whether these variations in $\mathcal{B}_0$ dominate over the changes in the geometry of topography and the vertical decay scale of mixing remains to be assessed with microstructure profiles reaching all the way to the seafloor and with theories that establish the relationship between $\mathcal{B}_0$, $\tan\theta$, d, and the deep ocean stratification. In this study (and in Ledwell's note) the stratification was assumed as known. However, Callies and Ferrari (2017, manuscript submitted to *J. Phys. Oceanogr.*) illustrate with a simple planetary geostrophic model that there are important relationships between the deep ocean stratification and the turbulence along ocean ridges and seamounts. Future work will need to establish whether those relationships have been violated in some of the diagnostic calculations of McDougall and Ferrari (2017) and Ledwell (2018).

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