On the Indeterminacy of Rotational and Divergent Eddy Fluxes*

BAYLOR FOX-KEMPER

Massachusetts Institute of Technology–Woods Hole Oceanographic Institution Joint Program in Oceanography and Ocean Engineering, Cambridge, Massachusetts

RAFFAELE FERRARI

Department of Earth, Atmospheric, and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, Massachusetts

JOSEPH PEDLOSKY

Department of Physical Oceanography, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts

21 May 2002 and 3 September 2002

ABSTRACT

The decomposition of an eddy flux into a divergent flux component and a rotational flux component is not unique in a bounded or singly periodic domain. Therefore, assertions made under the assumption of uniqueness, implicit or explicit, may be meaningless. Nondivergent, irrotational perturbations are allowed to any decomposition that may affect naive interpretation of the flux field. These perturbations are restricted, however, so that unique diagnostics can be formed from the flux field.

1. Introduction

Many authors have suggested that the relationship between the eddy flux and the large-scale gradient of a nearly conserved quantity (e.g., potential vorticity, temperature) is improved by dividing the eddy flux into purely divergent and purely rotational parts. This decomposition has a long history in electrodynamics (e.g., Griffiths 1998; Jackson 1998); it is relatively new to oceanography and meteorology.

The flux decomposition was introduced in this context to the geophysical community by Lau and Wallace (1979) in an attempt to show that the divergent portion of the flux is more aligned down the gradient of the mean field than the total flux. Previously, the wind itself had been decomposed from data (e.g., Sangster 1960). In this paper, we shall demonstrate the difficulty with such an attempt in a region with boundaries.

Helmholtz's theorem (e.g., Morse and Feshbach 1953) asserts that the decomposition into rotational and divergent fluxes is unique in an infinite domain, assuming the fluxes decay rapidly enough as infinity is approached (faster than r^{-2} in three dimensions, faster than r^{-1} in two; r is the distance from the origin). Morse and Feshbach assert that the proof of this theorem holds in a bounded domain as well, so long as the fluxes are taken to be zero everywhere outside of the domain. Though mathematically convenient, this reasoning uses information unavailable in the physical world.

In the physical world, we have information only about the flux to be decomposed and its boundary conditions. The divergent and rotational fluxes cannot be observed individually, and without using additional constraints these fluxes and their boundary conditions cannot be determined uniquely. Any physical theory ought not to depend on either of these components individually.

In this note, examples are used to demonstrate that there is no unique decomposition of an eddy flux into divergent and rotational parts in a bounded domain. In addition, an example is presented for a singly periodic domain. Last, some alternative diagnostics that provide information about the divergent and rotational parts of a flux are presented. Unlike the flux decomposition, these diagnostics are unique, so they are of greater potential use in the study of eddy flux parameterizations.

* Woods Hole Oceanographic Institution Contribution Number 10737.

Corresponding author address: Baylor Fox-Kemper, MIT Rm. 54-1715, 77 Massachusetts Ave., Cambridge, MA 02139.
E-mail: baylor@mit.edu

2. Analysis

Consider a vector field $\overline{\mathbf{v}'q'}$ intended to represent an eddy advection flux of a nearly conserved quantity q by the 3D flow $\mathbf{v}$. The overbar denotes an average over many realizations or over a time longer than eddy timescales, $\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}'$, and $q = \overline{q} + q'$. In this section, a method for dividing such a flux into divergent and rotational components is presented.

To construct a decomposition into rotational and divergent fluxes, one can use potentials: the divergent flux is a gradient of a scalar, and the rotational flux is a curl of a vector potential:

$$\overline{\mathbf{v}'q'} = \nabla\phi + \nabla \times \mathbf{A}, \quad (1)$$

$$\overline{\mathbf{v}'q'}_{\mathrm{rot}} = \nabla \times \mathbf{A}, \quad (2)$$

$$\overline{\mathbf{v}'q'}_{\mathrm{div}} = \nabla\phi. \quad (3)$$

The scalar potential is ϕ, and the vector potential is $\mathbf{A}$. Any continuous vector field can be represented by a scalar potential and a vector potential, but the choice of potentials is not unique. The resulting fluxes ($\overline{\mathbf{v}'q'}_{\mathrm{rot}}$ and $\overline{\mathbf{v}'q'}_{\mathrm{div}}$) are invariant under a gauge transformation of the vector potential ($\mathbf{A}' = \mathbf{A} + \nabla\lambda$) and to an addition of a constant to the scalar potential ($\phi' = \phi + C$). An additional indeterminacy is present if the boundary conditions on the scalar potential and each component of the vector potential are not known individually. This kind of indeterminacy will not affect the total flux, but it may affect the divergent and rotational fluxes.

We can calculate field equations for the potentials by taking derivatives of the known $\overline{\mathbf{v}'q'}$ field:

$$\nabla \cdot \overline{\mathbf{v}'q'} = \nabla^2\phi \quad \text{and} \quad (4)$$

$$\nabla \times \overline{\mathbf{v}'q'} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}. \quad (5)$$

The choice of a particular gauge sets the value of $\nabla \cdot \mathbf{A}$.

The boundary conditions for these field equations are not complete. The only physical variable is the total flux, so there are boundary conditions only on the total flux. A typical boundary condition is impermeability: $\hat{\mathbf{n}} \cdot \mathbf{v}' = 0$. There usually is also another boundary condition that makes the tangential components $\hat{\mathbf{t}}$ vanish as well (e.g., $q' = 0$, or no slip $\hat{\mathbf{t}} \cdot \mathbf{v}' = 0$). Thus, the boundary conditions on the decomposed fluxes are usually

$$\hat{\mathbf{n}} \cdot (\overline{\mathbf{v}'q'}_{\mathrm{div}} + \overline{\mathbf{v}'q'}_{\mathrm{rot}}) = 0 \quad (6)$$

$$\hat{\mathbf{n}} \times (\overline{\mathbf{v}'q'}_{\mathrm{div}} + \overline{\mathbf{v}'q'}_{\mathrm{rot}}) = 0, \quad (7)$$

where $\hat{\mathbf{n}}$ is the outward unit vector normal to the boundary.

For geophysical fluids, one is usually attempting this decomposition for fluxes within a two-dimensional surface, although the fluxes and potentials may depend on the third (vertical) dimension. For example, in quasigeostrophic and shallow water models, the fluxes of interest lie in the horizontal x, y plane although they are also functions of height z. For simplicity, in this note we restrict our examples to two dimensions, although the indeterminacy is also present in three dimensions. For two-dimensional fluxes, we need only consider the vertical component of the vector potential and choose to set its other components to zero. In a similar way, we consider only the horizontal components of the gradient of the scalar potential. Thus,

$$\overline{\mathbf{v}'q'} = \nabla_h\phi + \hat{\mathbf{k}} \times \nabla_h\chi, \qquad \chi \equiv -\hat{\mathbf{k}} \cdot \mathbf{A}, \quad (8)$$

where the subscript h indicates that derivatives are taken only in the horizontal, so that $\nabla_h = (\partial_x, \partial_y, 0)$. The field equations are

$$\nabla_h^2\phi = \nabla \cdot \overline{\mathbf{v}'q'} \quad (9)$$

$$\nabla_h^2\chi = \hat{\mathbf{k}} \cdot \nabla \times \overline{\mathbf{v}'q'}. \quad (10)$$

The boundary conditions are

$$\hat{\mathbf{n}} \cdot (\overline{\mathbf{v}'q'}_{\mathrm{rot}} + \overline{\mathbf{v}'q'}_{\mathrm{div}}) = \frac{\partial\phi}{\partial n} - \frac{\partial\chi}{\partial s} = 0 \quad (11)$$

and

$$\hat{\mathbf{n}} \cdot [\hat{\mathbf{k}} \times (\overline{\mathbf{v}'q'}_{\mathrm{rot}} + \overline{\mathbf{v}'q'}_{\mathrm{div}})] = \frac{\partial\phi}{\partial s} + \frac{\partial\chi}{\partial n} = 0, \quad (12)$$

where $\partial/\partial n$ is the derivative in the direction outward normal to the boundary, and $\partial/\partial s$ is the derivative in the direction along the boundary at a right angle to the left of $\hat{\mathbf{n}}$. The boundary conditions on the divergent and rotational fluxes individually are unknown; only the boundary conditions on the total flux are known.

Suppose the decomposition into divergent and rotational fluxes were unique. One could choose any boundary condition that was consistent with one of those above, for example $\partial\phi/\partial n = \chi = 0$. There is a choice of boundary condition because the problem for the potentials is only second order, requiring only a single boundary condition, while the total flux is consistent with both an impermeability boundary condition and a frictional one. With either choice, the field equations and the boundary conditions are set, and the divergent and rotational fluxes would be determined.

The decomposition is not unique, however. Consider a different decomposition, formed with potentials ϕ' and χ', that may individually satisfy different boundary conditions than the original ϕ and χ. The new fields will also obey Eqs. (9)–(12) if the perturbation potentials p and c, which are the difference between the two solutions, satisfy the following equations:

$$\nabla_h^2 p = \nabla_h^2 c = 0, \quad p \equiv \phi' - \phi, \quad c \equiv \chi' - \chi, \quad (14)$$

with boundary conditions

$$0 = \frac{\partial p}{\partial x} - \frac{\partial c}{\partial y} \quad (14)$$

$$0 = \frac{\partial p}{\partial y} + \frac{\partial c}{\partial x}. \quad (15)$$

There is an entire class of functions p and c that satisfy these equations. To generate a perturbation for any choice of ϕ and χ, one chooses p such that $\nabla_h^2 p = 0$ *with any boundary conditions whatever*, and then a c can be easily generated from p. In the next section, we give some examples of such perturbations.

The functions p and c must be harmonic within the domain—that is, $\nabla_h^2 p = \nabla_h^2 c = 0$—and satisfy the Cauchy–Riemann relations [Eqs. (14) and (15)] on the boundaries. Solutions to this problem are such that $p + ic$ is an analytic function of $z = x + iy$. The reverse is true as well; the analytic functions seem to exhaust the solutions of Laplace's equation [Eq. (13)] with Cauchy–Riemann boundary conditions [Eqs. (14) and (15)]. Thus, any analytic function $f(z)$ with no singularities in the domain is such that its real part represents the potential p and the imaginary part i represents the potential c. The choice of possible perturbations is otherwise unlimited. For example, we could choose $f(z) = z$ and find that $p = x$ and $c = y$ are solutions to Eqs. (13)–(15). More interesting examples are discussed in the next section.

The theory of analytic functions comes in handy in understanding the difference between the decomposition of the eddy fluxes in a finite domain and in an infinite domain. In a finite domain, there are no restrictions on possible singularities outside the domain, and the analytic function $f(z)$ can be nontrivial. In an infinite domain, we require that p and c be harmonic everywhere on the plane; thus $f(z)$ cannot have any singularity in the plane. If we further request that p and c decay fast enough to prevent singularities at infinity, then the choice is restricted to the only analytic function with no singularities: a constant. This constant is typically set to zero by requiring that p and c vanish at infinity. Regardless, a constant value in the potentials does not affect the fluxes.

Flux decomposition in a doubly periodic domain or on a sphere without continental boundaries is also unique. In these cases, there are no boundaries at which singularities can be concealed. Singularities in p and c within the domain are not allowed, because they will be discontinuous and their derivatives (the fluxes) will not exist. If we demand that p and c are continuous everywhere, the only solution is again $f(z) =$ constant.

Most geophysical problems have a boundary configuration in which flux decomposition is not unique. The only way that uniqueness can be obtained is to decompose the fluxes in a domain that has no boundary regions in which an arbitrary singularity in $p + ic$ can hide. Therefore, decompositions on the sphere, doubly periodic, and infinite domain without singularities at infinity are unique. In any domain with boundaries, however, no uniqueness is possible without additional constraints.

3. Example perturbations

A simple example of a perturbation to the potentials corresponds to the analytic function $f(z) = z - iz$:

$$p = x + y \tag{16}$$

$$c = -x + y. \tag{17}$$

The resulting change in the fluxes is

$$\Delta\overline{\mathbf{v}'q'}_{\text{div}} = (1, 1) \quad \text{and} \tag{18}$$

$$\Delta\overline{\mathbf{v}'q'}_{\text{rot}} = (-1, -1). \tag{19}$$

Thus, the perturbation to the divergent flux is constant, and the perturbation to the rotational flux is also constant. The two perturbations add up to zero as required. The existence of this perturbation is not really a surprise; one is accustomed to fluxes being specified only up to a meaningless constant.

A second example is less trivial, corresponding to the analytic function $f(z) = z^2$:

$$p = x^2 - y^2 \tag{20}$$

$$c = 2xy. \tag{21}$$

The change in the fluxes is

$$\Delta\overline{\mathbf{v}'q'}_{\text{div}} = (2x, -2y) \tag{22}$$

$$\Delta\overline{\mathbf{v}'q'}_{\text{rot}} = (-2x, 2y). \tag{23}$$

Consider the effect of this perturbation if the total flux were $\overline{\mathbf{v}'q'} = (-kx, 0)$. We see that, although one choice of the divergent flux is $\overline{\mathbf{v}'q'}_{\text{div}} = (-kx, 0)$, another equal ly valid choice would be $\overline{\mathbf{v}'q'}_{\text{div}} = [-(k - 2)x, -2y]$ with $\overline{\mathbf{v}'q'}_{\text{rot}} = (-2x, 2y)$. Therefore, if diffusivity were measured by comparing the divergent flux with a large-scale gradient, it could easily be in error in both magnitude and direction.

The perturbation potentials satisfy the Cauchy–Riemann relations not only at the boundary, but everywhere in the basin. Therefore, they apply to basins of any finite shape. Notice, though, that, in both of the examples presented so far, the perturbations are unbounded at infinity and therefore are excluded by the Helmholtz theorem. In a finite basin, however, there is no *physical* reason to eliminate either perturbation.

The perturbation to the boundary condition can be even more complicated. Consider decomposing the flux in a square two-dimensional basin with x and y ranging from 0 to 1. Choose any perturbation to the boundary condition on ϕ at the $y = 0$ boundary. We can solve for the perturbation throughout the basin by Fourier transform, producing the following:

$$p = \sum_{k=0}^{\infty} \left\{ a_k \sin(k\pi x) \frac{\sinh[k\pi(1 - y)]}{\sinh(k\pi)} \right\} \tag{24}$$

$$c = \sum_{k=0}^{\infty} \left\{ -a_k \cos(k\pi x) \frac{\cosh[k\pi(1 - y)]}{\sinh(k\pi)} \right\}, \tag{25}$$

where k is the index of the Fourier expansion, and the a_k are the Fourier coefficients found by Fourier transform of the perturbation along the boundary. As in the two preceding examples, this perturbation obeys the

Cauchy–Riemann equations everywhere, so it would be acceptable in any basin of finite dimension. The analytic function corresponding to this solution is

$$f(z) = \sum_{k=0}^{\infty} a_k \frac{\cos[k\pi(z - i)]}{\sin(ik\pi)}. \tag{26}$$

Because the equations on p and c are linear, similar solutions describing perturbations from the other boundaries may also be superposed.

This perturbation to the potentials works equally well in a domain with a periodic boundary at $x = 0$ and $x = 2$. The boundary is extended so that both p and c will be periodic in x. Note, however, that this perturbation to the fluxes would be discontinuous in y in a doubly periodic basin.

This example also illuminates the way that uniqueness is approached as one moves the boundaries farther and farther away toward the limit of the infinite domain. As the distance from the boundary is increased, the effects of singularities hidden in the boundaries diminish, because the harmonic field from a point singularity decays as r^{-1} in two dimensions, and the resulting fluxes from this point singularity will decay as r^{-2} away from the boundaries. Singularities with larger spatial structure will decay more slowly (e.g., the examples in this section decay more slowly because they require the domain to be surrounded by singularities), so the perturbations p and c can only be discounted for small-scale variations far away from the boundary. In the preceding Fourier transform example, one sees that the decay scale away from the boundary is the same as spatial scale along the boundary. If an additional constraint is added to the decomposed fluxes—for example, if they are bounded in magnitude at the boundary—then this decay could be used to show that the decomposed fluxes are approximately unique far away from the boundaries, but such an argument would require careful treatment.

So, there are infinitely many perturbations available. Figure 1 gives four different realizations of a divergent flux. All four are completely consistent with the same total flux.

The preceding examples demonstrate that the lack of uniqueness in bounded regions is due to the freedom to choose p arbitrarily along a boundary. In two or three dimensions, a perturbation can be constructed by choosing p to be any harmonic function, and then p can be used to generate the boundary conditions for the Laplace equation on the vector potential (which is simply c in the two-dimensional case). In two dimensions, the perturbation can be constructed by choosing any analytic function as $p + ic$. In the spirit of complex integration of analytic functions, the variations in p and c at the boundary can be considered as singularities anywhere outside the domain. If there are no boundaries in the problem to conceal singularities, and there are no singularities at infinity, only then is the decomposition unique.

4. Unique versus arbitrary diagnostics

The decomposition of an eddy flux into divergent and rotational fluxes is not unique in a bounded domain. However, the variation among the possible decompositions is limited. The perturbations to the fluxes must be irrotational and nondivergent, and they cannot contribute to the total flux. Thus the curl, divergence, and the value of total flux are all well defined. Some integrals of the fluxes are also unique. In this section we present some diagnostics of the fluxes that are unique because they are invariant under the perturbations described in the previous sections.

The flux divergence and the flux curl are excellent indicators of the physical import of the flux. It is these derivatives that appear in the equations for eddy interaction with the mean flow. However, it is often the case, both in numerical simulations and data analysis, that $\overline{\mathbf{v}'q'}$ is poorly resolved or poorly averaged ($\overline{\mathbf{v}'q'}$ often must be averaged over very long times to obtain a smooth field). When the $\overline{\mathbf{v}'q'}$ is too noisy to take sensible derivatives, the integrals of the divergence and curl are more useful diagnostic tools.

Integrals of the flux divergence over well-chosen areas reveal much about the divergent flux. Because of the divergence theorem, an area integral is an unambiguous method of determining the value of the divergent flux, not in one location, but on average around the bounding contour of the integration. For an area A bounded by the contour S,

$$\iint_A \nabla \cdot \overline{\mathbf{v}'q'}\, dx\, dy = \oint_S \overline{\mathbf{v}'q'} \cdot \hat{\mathbf{n}}\, dl$$
$$= \oint_S \overline{\mathbf{v}'q'}_{\text{div}} \cdot \hat{\mathbf{n}}\, dl, \tag{27}$$

where $\hat{\mathbf{n}}$ is an outward normal unit vector. A similar diagnostic also exists in three dimensions with volume integration of the region bounded by a surface. This integration produces a unique value *regardless of choice of* $\overline{\mathbf{v}'q'}_{\text{div}}$ from the infinite class described in the preceding sections. The contribution from the analytic function perturbations cancels out in the integral. Any area of interest can be chosen: areas with interesting forcing, entire oceans, straits of narrows, and so on. If the area of integration is chosen to be the area within a mean streamline, the average cross-streamline eddy flux is revealed. With the area of integration chosen to be within a $\overline{q}$ contour, the average eddy flux is assessed as up- or downgradient.

By integrating along a closed contour, the average rotational component of the flux can also be measured using Stokes's theorem. As in the case of the integral of the divergence, a similar diagnostic exists in three dimensions with volumes replacing areas and surface replacing contours. The same areas of integration sug-

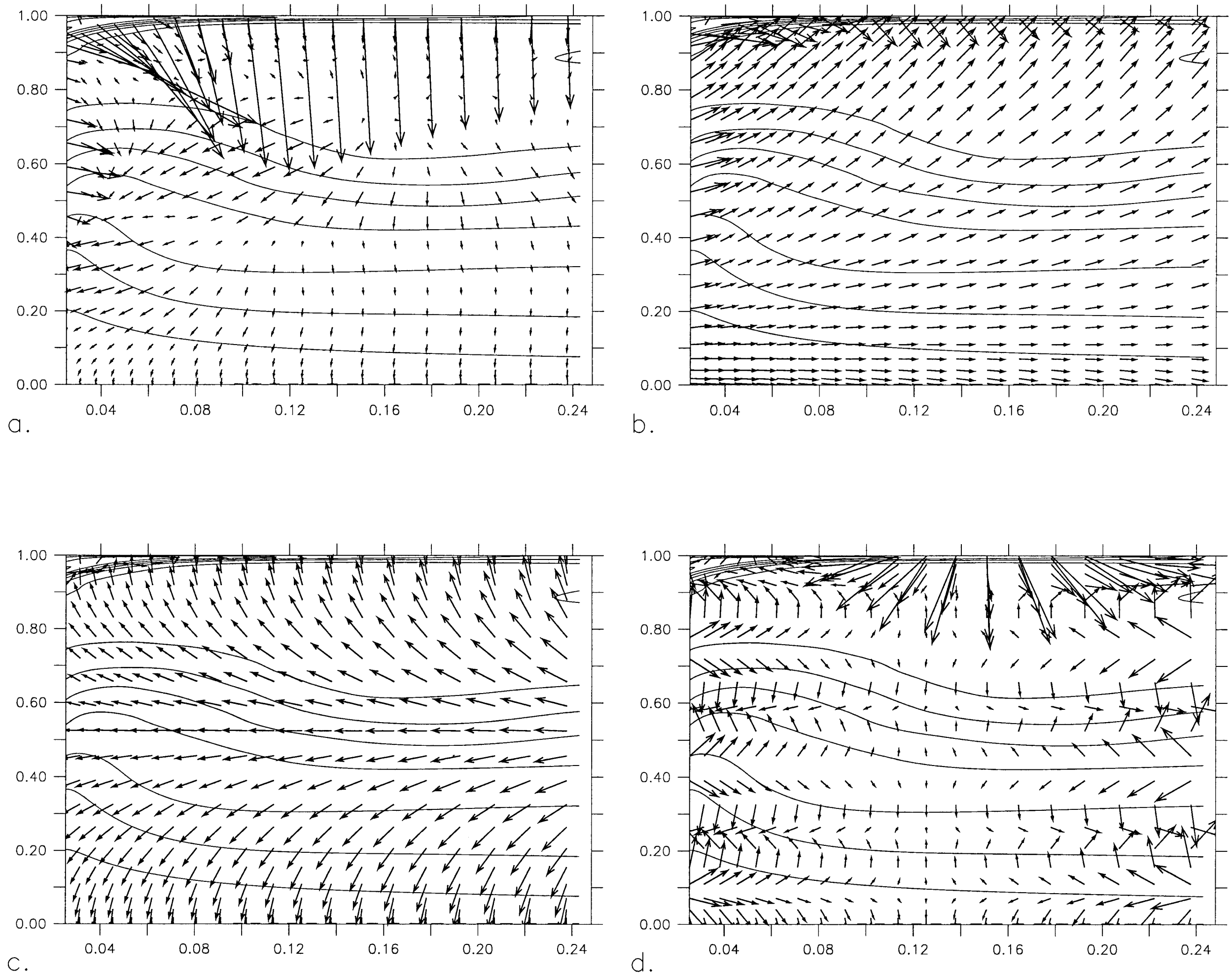

FIG. 1. These four flux fields depict four realizations of $\overline{\mathbf{v}'q'}_{\mathrm{div}}$ for the same hypothetical total flux. All four realizations have the same divergence and are irrotational. The $\overline{q}$ field is the potential vorticity in a region of a numerical model of barotropic flow in a basin with boundaries at $x = (0, 1)$ and $y = (0, 1)$. A region within this domain is selected because the mean field was interesting but not overly complicated. (a) Realization generated to be exactly down the mean gradient: $\overline{\mathbf{v}'q'}_{\mathrm{div}} = -0.2\boldsymbol{\nabla}\overline{q}$. The other three realizations are generated by the addition of harmonic function perturbations: (b) $p = y^2 - x^2 + 2x$, (c) $p = -\sin(\pi y)e^{\pi x}$, and (d) $p = \sin(10\pi x)e^{10\pi(y-1)} - \sin(6\pi y)e^{6\pi(x-1/4)} - \sin(6\pi y)e^{-6\pi x}$. The arrows representing the divergent flux have been rescaled for visualization in each figure.

gested in the preceding paragraph will serve here as well:

$$\iint_A \hat{\mathbf{k}} \cdot \boldsymbol{\nabla} \times \overline{\mathbf{v}'q'}\ dx\ dy = \oint_S \overline{\mathbf{v}'q'} \cdot \hat{\mathbf{t}}\ dl$$

$$= \oint_S \overline{\mathbf{v}'q'}_{\mathrm{rot}} \cdot \hat{\mathbf{t}}\ dl. \qquad (28)$$

The vector $\hat{\mathbf{t}}$ is unit length and tangent to the contour of integration, pointing in a counterclockwise sense.

As an example of how the integral constraints might be used to diagnose the effects of eddies, consider the double-gyre circulation of Marshall (1984). The integral of the eddy flux divergence over the whole basin will vanish, but the integral over only one of the two gyres will not. The amount by which the integral differs from zero will be the integral of the divergent intergyre eddy flux.

It is possible to construct other unique diagnostics addressing the divergent or rotational fluxes using the divergence theorem and Stokes's theorem. One can imagine correlations or other averages being constructed uniquely, but great care must be taken to heed the lack of uniqueness in any choice of flux decomposition.

Marshall and Shutts (1981) and Marshall (1984) generate a decomposition of physical relevance in the case where $q = q(\chi)$. This assumption allows them to single out a term in the eddy enstrophy equation $(1/2)(d\overline{\chi}/d\overline{q})\hat{\mathbf{k}} \times \boldsymbol{\nabla}\overline{q'^2})$. When $q = q(\chi)$, this term coincides with a choice of the rotational flux. This method gives immediate physical meaning to the choice of the rotational flux. This method has also been applied to oceanographic data (e.g., Cronin and Watts 1996). However, the approximation that $q = q(\chi)$ is sometimes inappropriate and depends strongly on the frictional boundary conditions used [as discussed in Roberts and Marshall (2000)].

Bryan et al. (1999) avoid the indeterminacy altogether by comparing the magnitude of the flux divergence and the magnitude of the total flux. They find that, in their calculations, these fields are correlated.

Lau and Wallace (1979) perform a hemispherical decomposition into the rotational and divergent parts. They acknowledge the indeterminacy in the boundary conditions and eventually set ϕ and χ to zero on their boundary at 20°N latitude. They assert that other boundary conditions were also used and that no significant change in the decomposition was observed, although the specifics of these calculations are not given.

Often other arguments are used to rationalize a particular choice out of the many possible decompositions into rotational and divergent fluxes. One can choose (arbitrarily) to impose the same boundary conditions on the divergent or rotational fluxes individually as apply to the total flux (e.g., Roberts and Marshall 2000). One can arbitrarily set the divergent fluxes to zero outside of the region in which they are known, as in Watterson (2001). Using inverse methods, one could determine the decomposition with the minimum integral of $|\overline{\mathbf{v}'q'}_{\text{div}}|^2$ or the decomposition with the divergent flux that is the "most downgradient" [the one minimizing the quantity $(\overline{\mathbf{v}'q'}_{\text{div}} + k\boldsymbol{\nabla}\overline{q}) \cdot (\overline{\mathbf{v}'q'}_{\text{div}} + k\boldsymbol{\nabla}\overline{q})$], and so on ad infinitum. Different physical interpretation will motivate different choices. It cannot be overemphasized that these represent choices of a flux field decomposition, not *the* flux field decomposition. If a unique decomposition is obtained with additional constraints, the physical relevance of a divergent or rotational part of the total flux relies upon the physical relevance of those constraints.

5. Conclusions

In this note, numerous examples have been given to show that the decomposition of a flux into divergent and rotational parts is not unique when boundaries are present. Just as the constant value of a potential has no physical meaning, the harmonic part of a potential leading to a divergent or rotational flux has no physical meaning in a bounded domain. If the flux decomposition is calculated, it is perilous to ignore this indeterminacy.

In construction of en eddy parameterization, it is sometimes useful to consider a solely divergent eddy flux field. However, the authors recommend that "downgradient" eddy parameterizations be considered not as representative of the divergent flux, but as a path to the divergence itself, which is unambiguous. As emphasized by Lorenz (1967), the divergence is the dynamically important quantity.

Because of the indeterminacy, use of the divergent or rotational fluxes as a diagnostic of the effects of eddies is extremely difficult. Because the divergent component of the flux is ambiguous, the validity of a parameterization can not be judged adequately by comparing a choice of divergent flux component directly with the flux from the parameterization. However, the unique diagnostics presented here are useful in assessing the validity of parameterizations and the effects of eddy fluxes on the mean fields.

REFERENCES

Bryan, K., J. K. Dukowicz, and R. D. Smith, 1999: On the mixing coefficient in the parameterization of bolus velocity. *J. Phys. Oceanogr.,* **29,** 2442–2456.

Cronin, M., and D. R. Watts, 1996: Eddy–mean flow interaction in the Gulf Stream at 68°W. Part I: Eddy energetics. *J. Phys. Oceanogr.,* **26,** 2107–2131.

Griffiths, D., 1998: *Introduction to Electrodynamics.* 3d ed. Prentice Hall, 576 pp.

Jackson, D., 1998: *Classical Electrodynamics.* 3d ed. John Wiley and Sons, 808 pp.

Lau, N. C., and J. M. Wallace, 1979: On the distribution of horizontal transports by transient eddies in the Northern Hemisphere wintertime circulation. *J. Atmos. Sci.,* **36,** 1844–1861.

Lorenz, E. N., 1967: *Nature and Theory of the General Circulation of the Atmosphere.* World Meteorological Organization, 161 pp.

Marshall, J. C., 1984: Eddy mean flow interaction in a barotropic model. *Quart. J. Roy. Meteor. Soc.,* **100,** 573–590.

——, and G. Shutts, 1981: A note on rotational and divergent eddy fluxes. *J. Phys. Oceanogr.,* **11,** 1677–1680.

Morse, P. M., and H. Feshbach, 1953: *Methods of Theoretical Physics.* McGraw-Hill, 1978 pp.

Roberts, M. J., and D. P. Marshall, 2000: On the validity of downgradient eddy closures in ocean models. *J. Geophys. Res.,* **105,** 28 613–28 627.

Sangster, W., 1960: A method for representing the horizontal pressure force without reduction of pressures to sea level. *J. Meteor.,* **17,** 166–176.

Watterson, I. G., 2001: Decomposition of global ocean currents using a simple iterative method. *J. Atmos. Oceanic Technol.,* **18,** 691–703.