Suppression of Eddy Diffusivity across Jets in the Southern Ocean

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ABSTRACT

Geostrophic eddies control the meridional mixing of heat, carbon, and other climatically important tracers in the Southern Ocean. The rate of eddy mixing is typically quantified through an eddy diffusivity. There is an ongoing debate as to whether eddy mixing in enhanced in the core of the Antarctic Circumpolar Current or on its flanks. A simple expression is derived that predicts the rate of eddy mixing, that is, the eddy diffusivity, as a function of eddy and mean current statistics. This novel expression predicts suppression of the cross-jet eddy diffusivity in the core of the Antarctic Circumpolar Current, despite enhanced values of eddy kinetic energy. The expression is qualitatively and quantitatively validated by independent estimates of eddy mixing from altimetry observations. This work suggests that the meridional eddy diffusivity across the Antarctic Circumpolar Current is weaker than presently assumed because of the suppression of eddy mixing by the strong zonal current.

1. Introduction

Geostrophic eddies generated by instabilities of large-scale ocean currents dominate the kinetic energy of the ocean and act to rapidly redistribute tracers such as heat and carbon. Ocean models used for climate studies are still too coarse to resolve the spatial scales at which geostrophic eddies develop and must resort to parameterize their effect. Eddy transport of tracers is typically parameterized assuming that the eddy tracer flux is related to the mean tracer gradient through an eddy diffusivity K. Arguably, the lack of knowledge of the spatial and temporal variations in the magnitude of K is a key limitation to the skill of ocean models.

Early estimates of eddy diffusivities were obtained from mooring time series and floats. Mooring data of time series of velocity $\mathbf{v}$ and temperature T were used to construct $\overline{\mathbf{v}'T'}$ and its relationship to $\nabla \overline{T}$ (e.g., Bryden and Heath 1985). However, Wunsch (1999) shows that the time series are too short (a few years at most) to obtain statistically representative results. Moreover, interpretation of such point estimates are seriously compromised by the presence of large nondivergent eddy fluxes that play no role in eddy–mean flow interactions (Marshall and Schutts 1981). The rate of dispersion of floats and drifters can also be related to an eddy diffusivity (e.g., Taylor 1921; Davis 1991; LaCasce and Bower 2000; Lumpkin and Pazos 2007). However, questions remain as to how such Lagrangian diffusivities are related to the Eulerian eddy diffusivities employed in large-scale ocean models.

Satellite altimetry provided the first global estimates of eddy statistics at the ocean surface. Holloway (1986), Keffer and Holloway (1988), and Stammer (1998) relied on mixing length theory (Prandtl 1925) to compute maps of eddy diffusivity from sea surface height variability. Mixing length theory suggests that in turbulent flows the eddy diffusivity can be expressed as $K \sim \upsilon \ell$, where υ is the rms eddy velocity and ℓ is a mixing length. The eddy velocity was estimated from gradients of surface height through the geostrophic relation. The mixing length was set proportional to the observed eddy size. The inferred K peaked in the core of strong currents, such as the western boundary currents and the Antarctic Circumpolar Current (ACC) of the Southern Ocean, where eddy velocities are largest.

Marshall et al. (2006) took a different approach to estimating K from altimetric measurements. They followed the technique pioneered by Nakamura (1996) to study eddy transport in the stratosphere. The idea is to numerically advect an idealized tracer using the geostrophic flow measured with the altimeter. The teased tracer filaments can be used to estimate the eddy diffusivity according to the formula $K = \mu L_{\text{contour}}^2 / L_0^2$, where

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μ is the background diffusivity used in the numerical code, L_{contour} is the observed length of a tracer contour twisted and folded by eddy stirring, and L_0 is the minimum (unstrained) length of the contour. Marshall et al. (2006) used the technique to estimate K across the ACC in the Southern Ocean (the technique cannot be used to estimate eddy mixing along mean currents). They found that K is largest on the equatorward flank of the ACC in the subtropics, whereas it is somewhat smaller in the core of the current. This pattern is opposite to that obtained in the studies reviewed above. Abernathey et al. (2010) suggested that the equatorward enhancement is associated with critical layers at the edge of the ACC, a result observed in atmospheric jets (Randel and Held 1991; d'Ovidio et al. 2009).

In this paper, we first discuss the differences between the spatial variations in K obtained by Keffer and Holloway (1988), Stammer (1998), and Marshall et al. (2006). Our focus is on the eddy transport across mean currents; eddy transport along mean currents is less important because the transport in that direction is dominated by mean current advection. We show that the cross-current effective diffusivity does indeed scale as $K \sim v\ell$, consistent with mixing length arguments. However, the calculation of the mixing length ℓ must be modified to account for the propagation of eddies, a pervasive feature of oceanic turbulence that is absent in classical three-dimensional turbulence (Chelton et al. 2007). Geostrophic eddies generated through baroclinic instabilities of a mean current (the case of eddies observed in the ACC) propagate against the surface current at a speed close to the speed of the mean current itself and they appear to be nearly stationary in the earth's reference frame. This upstream propagation reduces the across-current mixing length ℓ because the mean current advects tracer out of the eddy before much filamentation has occurred. This modulation was ignored in the studies by Holloway (1986), Keffer and Holloway (1988), and Stammer (1998), but it is crucial to interpret the diagnostic estimate of Marshall et al. (2006). In this paper, we show how to extend mixing length arguments to account for the suppression of ℓ by eddy propagation, and we explain why the effect is most prominent in the core of mean currents. Using altimetric data, we find that the mixing length suppression is so large that K is often smaller within the ACC than on its flanks, despite the rms eddy velocity v being largest in the ACC.

The paper is organized as follows. Section 2 derives an analytical expression for the eddy diffusivity that includes the effects of both transient eddies and mean flows. Section 3 describes the Southern Ocean altimetric data used to test the theory developed in section 2. Section 4 presents estimates of K obtained with Nakamura's technique applied to the altimetric data. The calculation is repeated for both a small sector in the Pacific and the whole Southern Ocean. It is shown that our analytical expression for the eddy diffusivity reproduces accurately the results of the Nakamura calculation for both regions considered. Section 5 offers conclusions.

2. Eddy diffusivity across jets

A property of geostrophic eddies is to greatly enhance the mixing of tracers. This enhanced mixing is generally cast as an eddy diffusivity that describes the enhancement of tracer flux over the diffusive molecular flux. The concept of eddy diffusivity is justified by appealing to an analogy between turbulent eddies and molecular diffusion: turbulent eddies move tracer parcels in erratic motions, much like bombardment by molecular agitation, and thus the action of turbulence may be represented as an enhanced diffusion. This is the essence of mixing length theory (Prandtl 1925). Papanicolaou and Pironneau (1981) showed that the analogy holds as long as there is a clear separation between the spatial and temporal scales of the eddies and the large-scale circulation. Even in this limit, however, the analogy is not perfect. Eddy diffusivities, unlike molecular diffusivities, can be strongly modulated by variations in the large-scale currents. Such modulations are often ignored in the oceanographic literature, but they are a crucial feature of eddy mixing (e.g., Andrews et al. 1987). The goal of this section is to illustrate the relationship between mean currents and eddy diffusivity and to derive an expression for the eddy diffusivity that captures this effect. In the remaining sections, we test the expression for the eddy diffusivity against altimetric data.

We focus our analysis on the ACC system in the Southern Ocean, which provides a natural setup to test our theory of eddy transport across permanent currents. However, we expect our results to apply equally well to tracer transport across any permanent current system, such as the western boundary currents and equatorial currents. As mentioned in the introduction, our focus is on cross-current eddy transport, because along-current transport is dominated by the mean flow and not by transient eddies.

Observations (e.g., Orsi et al. 1995; Tulloch et al. 2009a, manuscript submitted to *J. Phys. Oceanogr.*) and numerical models (e.g., Hallberg and Gnanadesikan 2006) show that the ACC system is best described as the superposition of permanent jets and propagating eddies. Following a "Reynolds decomposition" of variables into an along-jet average and departures from that average, the cross-jet eddy diffusivity $K_{\perp}$ relates the turbulent flux of a tracer $\overline{v'c'}$ across the jet to the mean tracer gradient across the jet, $\partial \overline{C}/\partial y$,

$$\overline{v'c'} = -K_\perp \frac{\partial \overline{C}}{\partial y}. \qquad (1)$$

The overbar denotes the streamwise and temporal mean, and primes are the deviations from that mean. The eddy diffusivity $K_\perp$ is typically many orders of magnitude larger than the molecular value, implying that eddies enhance the flux of tracer across the jet. Herein, we use the symbol $K_\perp$ to refer to the eddy diffusivity across the mean streamlines.

There is a large literature on cross-jet tracer transport induced by small amplitude waves propagating along a jet. This body of work is generally referred to as "chaotic mixing" (e.g., Pierrehumbert 1990; Wiggins and Ottino 2004). The approach is purely kinematic; that is, the velocity field is prescribed and the focus is on the tracer transport that ensues. For our purposes, the most pertinent piece of chaotic mixing literature concerns a straight jet perturbed by steady waves (e.g., Bower 1991; Samelson 1992; Pratt et al. 1995; Yuan et al. 2004; Rypina et al. 2007). If the waves are of sufficiently large amplitude, the mean streamlines can close into recirculating regions bounded by a separatrix. Tracer parcels would be trapped forever in such regions. However, the tiniest additional time fluctuations in any one of the waves is sufficient to generate a stochastic band surrounding the separatrix where tracer parcels are chaotically mixed in and out of the recirculating region. Chaotic mixing does not occur if the waves are of small amplitude. In this limit the jet acts as a barrier to transport. Theorems have been proven to identify regions where chaotic mixing or barrier regions arise. While this theory is useful to study tracer transport in idealized flows, the most fundamental question in the oceanographic context is what sets the properties of the waves and the mean currents. Hence, we depart from the kinematic approach and derive a simple dynamical result that captures the essence of the eddy–mean flow interactions.

a. The surface quasigeostrophic model

Permanent currents and geostrophic eddies in the ocean have characteristic scales somewhat larger than the first deformation radius (e.g., Chelton et al. 2007; Tulloch et al. 2009b). At these scales, the dynamics is well described by the quasigeostrophic (QG) approximation. Hence, we will consider a QG model that captures some of the essential features of jets and eddies observed in the ACC system. This simple model will then be used to derive an expression for the eddy diffusivity.

Lapeyre and Klein (2006) have recently shown that QG models with potential vorticity (PV) gradients confined to the surface reproduce quite accurately the vertical and horizontal structure of ocean eddies in the upper

few hundred meters of the oceans. Isern-Fontanet et al. (2008) confirmed that such models reproduce accurately upper-ocean eddy statistics estimated from a primitive equation model of the North Atlantic. The reason for the skill is that surface and interior PV anomalies are strongly correlated and hence much of the eddy structure can be inferred from surface PV.

Potential vorticity gradients are, indeed, surface intensified in the ACC, and the approach of Lapeyre and Klein (2006) seems sensible. A QG model with PV gradients confined to the surface is essentially Eady's (1949) model without a lower boundary. It was first introduced by Held et al. (1995) and is often referred to as surface QG. We will herein use the surface QG model to illustrate mixing across the ACC. However, we wish to emphasize that our final results on eddy mixing are independent of the QG model used and apply to any geophysical flow supporting waves.

The surface QG model consists of two equations, a prognostic equation for the surface buoyancy and a diagnostic equation for the interior PV:

$$\partial_t b + J(\psi, b) = 0, \quad b = f\partial_z \psi \quad \text{at} \quad z = 0 \qquad (2)$$

and

$$\partial_x^2 \psi + \partial_y^2 \psi + \frac{f^2}{N^2}\partial_z^2 \psi = 0 \quad \text{for} \quad z < 0, \qquad (3)$$

where ψ is the geostrophic streamfunction, b is the surface buoyancy, and J is the Jacobian operator. The inertial, f, and stratification, N, frequencies are assumed to be constant. The first equation is the advection of surface PV, which is linearly proportional to surface buoyancy b in QG. The second equation states that the interior PV is uniform.

The dynamics is now decomposed into a broad jet in thermal wind balance and small-scale eddies. An expansion in slow/large and fast/short variables would result in the same equations. The structure of the mean jet is given by

$$U(z) = U_0 \frac{z + H}{H}; \quad \Psi(y, z) = -U(z)y;$$

$$B(y, z) = -\Gamma y + N^2 z, \qquad (4)$$

where $U(z)$ is a sheared jet in thermal wind balance with a constant lateral buoyancy gradient $\partial_y B = -\Gamma$ with $\Gamma = fU_0/H$. The jet has a magnitude U_0 at the surface at $z = 0$ and decreases to zero at a depth $z = -H$, but it does not vary in the horizontal to ensure that there is a clear horizontal scale separation between the mean flow and the eddies.

The perturbations from the mean gradient, that is, the eddies, satisfy the equations

$$b_t + J(\psi - U_0 y, b - \Gamma y) = 0; \quad \partial_x^2 \psi + \partial_y^2 \psi + \frac{f^2}{N^2} \partial_z^2 \psi = 0, \tag{5}$$

where b and ψ now represent the eddy fluctuations. To simplify the problem, the nonlinear term $J(\psi, b)$ is represented with a fluctuation–dissipation stochastic model. There is an extensive literature on stochastic models of turbulence (e.g., Farrell and Ioannou 1993; DelSole 2004). Here we follow the approach described in Flierl and McGillicuddy (2002) and write

$$\partial_t b + U_0 \partial_x b - \Gamma \partial_x \psi = \mathcal{U} \sqrt{\gamma} \, \text{Re}[r(t) e^{i(kx+ly)}] - \gamma b. \tag{6}$$

The fluctuations are generated with a white noise random process $r(t)$; that is, a process with zero mean and autocorrelation function $\langle r(t) r^*(t') \rangle = \delta(t - t')$, where angle brackets denote the expected value and the asterisk the complex conjugate.[1] The forcing is monochromatic to keep the problem linear and crudely represents the excitation of waves at the energy containing scale (k, l). Dissipation is through linear damping at a rate γ and mimics the damping of each wave through interaction

with other waves or through dissipation mechanisms. The constant $\mathcal{U}$ sets the amplitude of the equilibrated eddy field. DelSole (2004) presents compelling evidence that stochastic models, such as the one used here, have significant skill in predicting the structure of the eddy fluxes. However, for present purposes, the choice of a stochastic model is purely pedagogical. Other approaches, such as quasi-normal eddy damped Markovian models of turbulence (Holloway and Kristmannsson 1986) and homogenization theory for diffusion in periodic flows (Majda and Kramer 1999), lead to the same prediction for eddy diffusivities, but they involve more complex mathematics. Equation (6) should be viewed as a simple toy model that mimics some properties of geostrophic eddies in the Southern Ocean: (i) the leading order eddy velocity field is horizontally divergenceless, (ii) the correlation function of the eddy velocity field decays with time at a rate γ^{-1}, and (iii) eddies propagate along the zonal current.

Equation (6), together with the PV equation in (3), can be solved as described in the appendix. The solution for the streamfunction takes the form

$$\psi = \frac{\mathcal{U}}{\kappa} \sqrt{\gamma} \, \text{Re} \int_0^\infty r(t - \tau) \exp\left[i(kx + ly - kc_w \tau) - \gamma\tau + \frac{N\kappa}{f} z\right] d\tau; \tag{7}$$

that is, ψ is the superposition of stochastically excited surface edge waves propagating at a phase speed c_w,

$$c_w = \left(1 - \frac{\kappa_d}{\kappa}\right) U_0; \quad \kappa_d = \frac{f}{NH}, \tag{8}$$

with $\kappa^2 = k^2 + l^2$. The eddy kinetic energy is computed from (7),

$$\text{EKE} = \frac{1}{2}\langle u^2 + v^2 \rangle = \frac{1}{2}\langle |\nabla\psi|^2 \rangle = \frac{1}{4}\mathcal{U}^2 \exp\left(2\frac{N\kappa}{f} z\right). \tag{9}$$

The EKE is proportional to $\mathcal{U}^2$ and decays exponentially with depth. The literature on stochastic models at this point would look for a closure relationship between $\mathcal{U}^2$ and mean variables, such as the mean flow, the mean shear, the bottom drag, etc. However, our goal is to derive an expression of eddy diffusivity that can be directly

estimated from data. Altimetric data provide estimates of both $\mathcal{U}^2$ and U_0, and there is no need for a closure relationship.

Eady (1949) showed that baroclinic instability develops in the surface QG problem if one adds a rigid boundary at the bottom. Baroclinic eddies develop most rapidly at wavenumber $\kappa = 1.6 f/NH$ if the bottom boundary is at $z = -H$ (otherwise, H must be replaced by the channel depth). The eddies propagate against the mean flow so that their overall phase speed is somewhat smaller than the mean flow speed at the surface, $c_w = (1 - \alpha)U_0$, and is independent of scale. If the bottom boundary is at $z = -H$, then $\alpha = \frac{1}{2}$ but $\alpha > \frac{1}{2}$ in a deeper domain. In fully nonlinear surface QG solutions, the eddies become somewhat larger than the most unstable scale, as a result of an inverse energy cascade, and continue to propagate zonally. This scenario describes quite accurately what is observed in the Southern Ocean from altimeters: the energy containing eddies have scales two to three times larger than the most unstable scale (Scott and Wang 2005; Chelton et al. 2007; Tulloch et al. 2009b) and propagate downstream the ACC at a rate significantly slower (20%) than that of surface currents (Smith and Marshall 2009). The surface flow associated with the ACC is directed eastward, peaking at a speed of about 15 cm s^{-1}, whereas

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[1] This is equivalent to setting $r(t) = R \exp[i\theta(t)]$ with $\theta(t)$ random. If one generated the stochastic process numerically with a discrete time step dt, then R would have a magnitude inversely proportional to the time step dt and $\langle r(t) r^*(t') \rangle$ would be a box of width dt centered on $t = t'$ with height $1/dt$. In the continuous limit $dt \to 0$ and $R \to \infty$ give rise to the δ function.

the phase speed of the eddies is $c_w \sim 2$ cm s^{-1} and also directed eastward over the latitudinal range of the ACC (roughly 45°–55°S). This implies that the intrinsic phase speed of the eddies ($c_w - U_0$) is directed westward and close to the mean current speed. Smith and Marshall (2009) and A. C. Naveira Garabato et al. (2010, unpublished manuscript) further show that the eddy phase speed covaries with the mean current speed in the ACC latitude band, supporting our ansatz that $c_w = (1 - \alpha)U_0$.

North of the ACC the waves travel from east to west, as is found in the rest of the oceanic basins (Chelton and Schlax 1996). Both waves and mean flow speed are substantially weaker than in the ACC. Indeed, the mean currents are so weak that their contribution to the potential vorticity through their vertical shear is negligible. The growth rate of baroclinic instability is also small, being proportional to the current shear. At these latitudes eddy variability is associated with Rossby waves triggered by meteorological forcing and remote instabilities propagating along the planetary vorticity gradient. These effects could be included in the stochastic model by adding a planetary potential vorticity gradient, but we chose not to because the phase speed of Rossby waves is too weak to substantially affect eddy mixing in the latitude range we are considering.

The stochastic model used to describe eddy mixing across a jet relies on three drastic assumptions: 1) the eddy motions can be represented with a linear stochastic model, 2) the eddy forcing consists of a single wave with specific wavenumber, and 3) the eddy scale is much smaller than the mean current width. The question is whether the expression for the eddy diffusivity derived in the next section depends crucially on these assumptions. The choice of a stochastic model is one of simplicity. Similar expressions for the eddy diffusivity across a jet are derived in Majda and Kramer (1999) using homogenization theory for periodic flows and in Holloway and Kristmannsson (1986) using quasi-normal eddy damped Markovian closure for turbulent flows. Holloway and Kristmannsson further show that our results can be extended to multichromatic forcing, as we discuss in the next section. Hence, the first two assumption made to derive a simple model do not appear to limit its applicability to the study of eddy mixing across jets.

The assumption of a scale separation between eddy and mean flows is more problematic. While a scale separation appears to exist in some sectors of the Antarctic Circumpolar Current, there are many sectors where the mean jet width is as narrow as one eddy scale. Studies of mixing across idealized narrow parallel jets (e.g., Rypina et al. 2007) show that the key prediction of the stochastic model, that is, that mixing is suppressed across jets, continues to hold even in the absence of scale separation.

However, this cannot be proved in general. Alternatively, we will check whether the scale separation assumption is crucial to our results by testing the prediction of the SQG model versus independent estimates of mixing in the Southern Ocean. In section 4, we show that the model captures, qualitatively and quantitatively, the rates of mixing across the Antarctic Circumpolar Current and we will conclude that the scale separation assumption does not severely limit the application of our model to the real ocean.

b. Eddy diffusivity across a jet

In a seminal paper, Taylor (1921) showed that the eddy diffusivity for a homogeneous, isotropic turbulent field is proportional to the EKE times an eddy decorrelation time scale; that is, $K \propto \text{EKE} \; \gamma^{-1}$. This result is equivalent to Prandtl's (1925), if one sets the mixing length scale $\ell \propto \text{EKE}^{1/2} \; \gamma^{-1}$ and $K \propto \text{EKE}^{1/2}\ell$. Taylor's formula is often used to estimate eddy diffusivities in the ocean (e.g., Davis 1991; Stammer 1998). We show here that the mixing length scale, and hence K, is strongly modulated by the presence of a mean flow. This modulation is ignored in many oceanographic estimates of eddy diffusivities.

To compute an eddy diffusivity we first solve for the concentration of a passive tracer stirred by the surface QG streamfunction ψ in (7). This approach differs from that taken in many papers on eddy transport in that we use a well-posed dynamical model to derive ψ, instead of prescribing an arbitrary ψ and then inferring the associated transport properties (e.g., Wiggins and Ottino 2004). Let us consider a tracer with a large-scale linear gradient $\Gamma_c y$ and perturbations c generated by eddy stirring of that gradient,

$$\partial_t c + J[\psi - U(z)y, \, c] = -\Gamma_c \partial_x \psi, \qquad (10)$$

with ψ and $U(z)$ given in (7) and (4). In the appendix, we derive the solution for c and then compute the tracer flux across the jet $U(z)$ by taking the expected value of vc,

$$\langle vc \rangle = -\left\{ \frac{1}{4} U^2 \frac{k^2}{\kappa^2} \frac{\gamma}{\gamma^2 + k^2[c_w - U(z)]^2} \exp\left(2\frac{N\kappa}{f}z\right) \right\} \Gamma_c. \tag{11}$$

The eddy diffusivity is the ratio of the tracer flux and the mean tracer gradient and is given by

$$K_\perp = \frac{k^2}{\kappa^2} \frac{\gamma}{\gamma^2 + k^2[c_w - U(z)]^2} \text{EKE}, \qquad (12)$$

where we used (9) to express $K_\perp$ in terms of EKE. Interestingly, the eddy diffusivity depends not only on

EKE, γ, and the eddy scale through (k, l) but also on the propagation speed of the eddies c_w relative to the mean flow $U(z)$.

To make contact with mixing length theory, we can express $K_\perp$ in (12) as the product of the rms eddy velocity and a mixing length; that is, $K_\perp = \text{EKE}^{1/2}\ell$, where the mixing length is defined as

$$\ell = \frac{k^2}{\kappa^2}\frac{\gamma}{\gamma^2 + k^2[c_w - U(z)]^2}\text{EKE}^{1/2}. \quad (13)$$

The expressions for $K_\perp$ and ℓ have a very simple physical interpretation. When eddies propagate at the same speed as the jet and $c_w = U(z)$, then $\ell = k^2\kappa^{-2}\,\text{EKE}^{1/2}\,\gamma^{-1}$ and the expression for $K_\perp$ reduces to that derived by Taylor (1921). When the eddy phase speed does not match the jet velocity, the mixing length is reduced. This suppression is a purely kinematic effect. Advection by the mean jet peels tracer filaments out of an eddy before much stirring has occurred. The suppression is strong if the tracer is advected out of the eddy much faster than the decorrelation time scale of the eddy itself, $k^2(c_w - U(z))^2 \gg \gamma^2$.

The literature on wave–mean flow interactions possibly overemphasizes the importance of critical layers, where $c_w = U(z)$, as key regions for eddy mixing (e.g., Andrews et al. 1987). This literature takes a linear view and eddies are represented as periodic waves with an infinite decorrelation time scale, a limit corresponding to setting $\gamma = 0$ in our problem. In this limit, the eddy diffusivity vanishes everywhere except at critical layers. This view is somewhat misleading when interpreting mixing in real geophysical systems. Eddies do decorrelate over some finite time scale γ^{-1} through nonlinear interactions with other eddies, and the eddy diffusivity never vanishes. Critical layers are simply regions where suppression of ℓ by the mean flow is weak. They are not central to the discussion of eddy mixing.

The expression for the eddy diffusivity in (13) is best expressed as a modification of the diffusivity K_0,

$$K_\perp = \frac{K_0}{1 + \gamma^{-2}k^2[c_w - U(z)]^2}; \quad K_0 = \frac{k^2}{\kappa^2}\text{EKE}\gamma^{-1}, \quad (14)$$

where K_0 is the Taylor expression for north–south eddy diffusivity in the limit where eddies do not propagate with respect to the mean flow. This expression predicts that, at the surface, the eddy diffusivity is suppressed as a result of eddy propagation. This suppression can disappear below the surface if there are deep critical layers where $c_w = U(z)$. An enhancement of eddy diffusivity at depth has been observed in idealized numerical simulations of

channel flows (McWilliams and Chow 1981; Treguier 1999). More recently, Abernathey et al. (2010) diagnosed $K_\perp$ from a state estimate of the Southern Ocean and report enhanced values of diffusivity in correspondence of deep critical layers in the vertical. The expression (14) is also consistent with the results of Greenslade and Haynes (2008), who studied eddy transport across an atmospheric jet. In the upper parts of the flow, where the mean jet is strong and $c_w \ll U_0$, $K_\perp$ is suppressed while in the lower part of the flow, where the jet is weak and $c_w \approx U_0$, $K_\perp$ is enhanced.

The expression for the eddy diffusivity in (14) is very general and does not depend on the details of the surface QG model used to derive it. It is straightforward to extend the fluctuation–dissipation stochastic model to barotropic and baroclinic QG models. The only difference is that the edge waves supported by the surface buoyancy gradient in surface QG are replaced by barotropic and baroclinic Rossby waves supported by interior PV gradients. The expression for $K_\perp$ remains the same as in (14), but the wave phase speed is that associated with the vertical mode considered; that is, $c_w = U(z) - \beta/(k^2 + l^2 + k_n^2)$, where k_n is the inverse deformation radius corresponding to the mode considered (Pedlosky 1987). The expression in (14) can also be extended to multichromatic wave fields, as done, for example, in Holloway and Kristmannsson (1986). The result is an integral over all wavenumbers (k, l) of the expression in (14). In regions of strong eddy activity, such as the ACC, the kinetic energy, as estimated from the altimeter, has a well-defined peak. Hence, the integral is dominated by the scale corresponding to the peak in energy, and the expression for $K_\perp$ reduces to the monochromatic formula in (14).

Notice that the derivation presented here is not a full theory for the eddy diffusivity. Such a theory would require prognostic equations for the EKE and the decorrelation time scale γ^{-1}. Examples of such theories abound in the literature: Held and Larichev (1996), Visbeck et al. (1997), Lapeyre and Held (2003), Thompson and Young (2007), and Cessi (2008) are useful recent references. In these works, scaling arguments are presented to express EKE and γ (or equivalently ℓ) in terms of large-scale quantities such as the mean velocity, the mean potential vorticity, and their gradients. Interestingly, none of these theories discusses the role of eddy propagation on the rate of mixing. This is, instead, the focus of this work. We show that the speed of propagation of eddies modulates the eddy diffusivity and must be included in theories of eddy mixing. In the next section, we confirm that eddy fields with identical statistics, but different propagation speeds, result in different mixing rates. We suspect that this effect would not be captured in any of the aforementioned theories.

c. Estimating eddy diffusivity across jets from altimetry

Next, we simplify the expression for $K_\perp$ in (14) so that it depends only on quantities available from altimetric measurements. The analysis that follows applies at all depths in the ocean, but we present the results for the ocean surface where altimetric data are available.

The decorrelation time scale γ^{-1} represents the eddy interaction time scale in the stochastic surface QG model; that is, the time scale over which energy is transferred from the energy containing wave (k, l) to other waves. The eddy interaction time scale is proportional to the eddy strain rate $(\kappa^2 EKE)^{-1/2}$ in a turbulent field (e.g., McComb 1990; Salmon 1998; DelSole 2004). For our problem, we can therefore write

$$\gamma = d_0^{-1}\sqrt{\kappa^2 EKE}, \tag{15}$$

where d_0 is a constant proportionality coefficient. Such a relationship is also broadly supported by altimetric observations in the ocean, as discussed in section 3. Note that this relationship does not provide a closure for γ in terms of mean quantities; rather it relates γ to variables accessible from altimetry. Using the relationship (15), the expression for $K_\perp$ in (14) at $z = 0$ reduces to

$$K_\perp = \frac{K_0}{1 + 0.5d_0^2(c_w - U_0)^2/EKE}; \quad K_0 = \frac{1}{2}d_0\kappa^{-1}EKE^{1/2}, \tag{16}$$

where we further assumed that the energy containing eddies are isotropic and set $\kappa^2 = 2k^2$.

Next, we assume that the phase speed of eddies is proportional to the local mean current speed and set $c_w = (1 - \alpha)U_0$ because we have already mentioned that such a relation is at least crudely supported by altimetric observations (Smith and Marshall 2009). Note that this relation eliminates the possibility of critical layers at the surface where $c_w = U_0$. In a baroclinically unstable jet, eddies are generated with a phase speed slower than the mean current, and critical layers can form only if eddies propagate large distances across the mean current. Although we cannot exclude that eddies meander substantially across latitude bands, we press on and substitute $c_w = (1 - \alpha)U_0$ in (16). This relation is justified a posteriori by showing that the resulting expression for $K_\perp$ reproduces observed patterns of mixing,

$$K_\perp = \frac{K_0}{1 + d_2 U_0^2/EKE}; \quad K_0 = \frac{1}{2}d_0\kappa^{-1}EKE^{1/2}, \tag{17}$$

where $d_2 = 0.5\alpha^2 d_0^2$.

The mean and kinetic energies can be computed from altimetric data. What about K_0? Here K_0 represents the eddy diffusivity in a turbulent field where eddies are stationary. Holloway (1986) and Keffer and Holloway (1988) showed how to derive maps of K_0 from altimeter maps of sea surface height variability. The geostrophic relation links the surface velocity field to sea surface height variations h: $(u, v) = f^{-1}g(-\partial_y h, \partial_x h)$, where g is the acceleration of gravity. In our model, the EKE is related to fluctuations in h as $EKE = 0.5f^{-2}g^2\kappa^2\overline{h'^2}$, where h' is the eddy departure of h from its long-term mean $\overline{h}$. Substituting this expression in the definition of K_0 in (17), we have

$$K_0 = d_1\frac{g}{|f|}(\overline{h'^2})^{1/2} \tag{18}$$

with d_1 a new constant proportionality coefficient.[2]

Finally, we can combine (17) and (18) to obtain

$$K_\perp = d_1\frac{g}{|f|}\frac{(\overline{h'^2})^{1/2}}{1 + 2d_2|\nabla\overline{h}|^2/\overline{|\nabla h'|^2}}, \tag{19}$$

where we expressed the ratio of mean to eddy kinetic energy in terms of surface height, $U_0^2/EKE = 2|\nabla\overline{h}|^2/\overline{|\nabla h'|^2}$. The two constant coefficients d_1 and d_2 are estimated below fitting the expression in (19) to observational estimates of $K_\perp$. Keffer and Holloway (1988) showed that (19) is a good predictor of $K_\perp$ in the absence of eddy propagation and mean flows. Our goal is to test whether (19) captures the modulations due to the presence of jets in the ACC.

3. Data

Estimates of the geostrophic streamfunction at the ocean surface are the key ingredient to computing surface eddy diffusivities. We use sea level anomaly maps from the combined processing of Ocean Topography Experiment (TOPEX)/Poseidon and *European Remote Sensing Satellite-1* (*ERS-1*) and *ERS-2* altimetry data taken every 10 days with a spatial resolution of ¼°, which is sufficient to resolve mesoscale eddies with scales of 1°–2°. Sea level anomalies are computed with respect to a 3-yr mean (from January 1993 to January 1996). Additional information about the altimetric data

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[2] The proportionality coefficient d_1 relates K_0 and sea surface height displacements. The proportionality coefficient d_0 relates K_0 and the rms EKE. For the monochromatic eddy field used in our simple theory, $d_1 = d_0/2^{3/2}$; however, for a broad spectrum eddy field as observed in the ocean the two coefficients are not necessarily related.
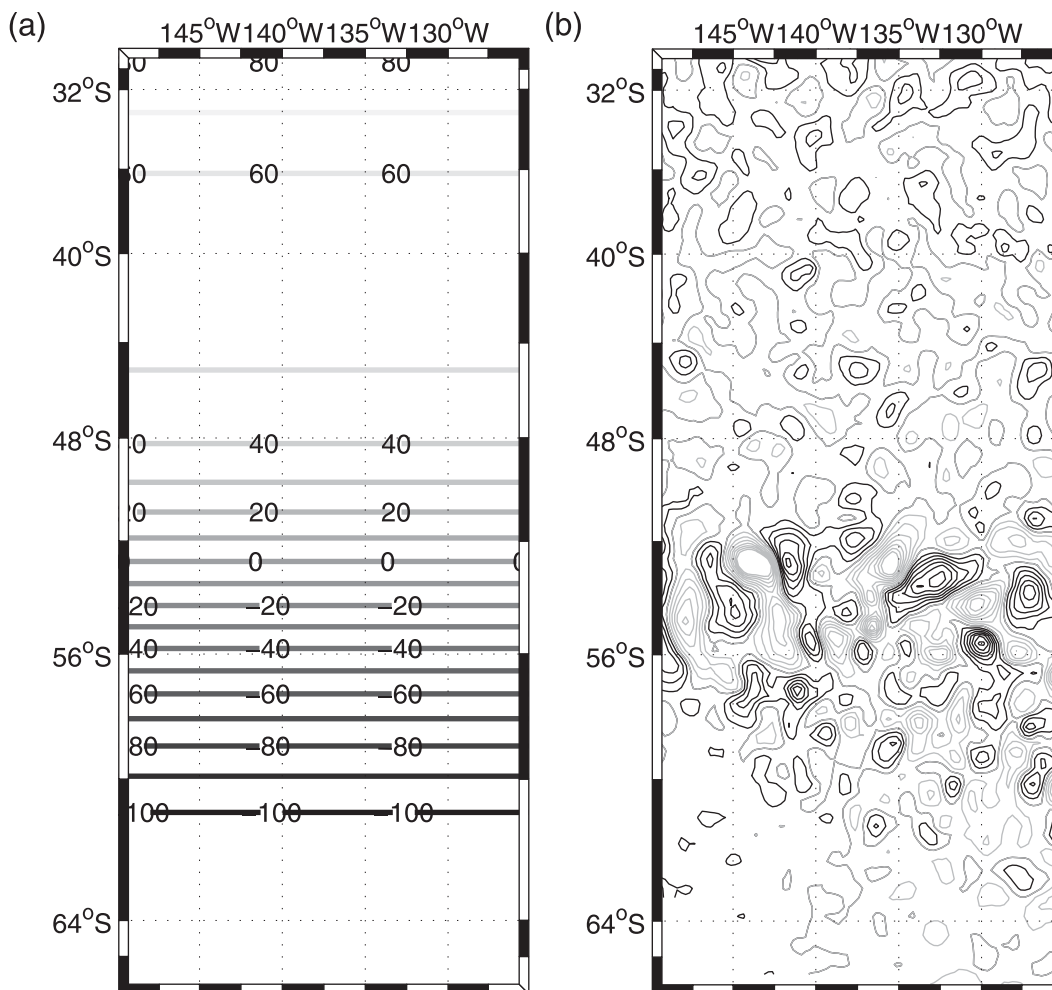
FIG. 1. (a) Zonally averaged mean sea surface height (cm) from a patch in the Pacific sector of the Southern Ocean between 66° and 30°S, 125° and 150°W. The contour interval is 10 cm from −100 to 80 cm. (b) Snapshot of the sea surface height anomaly in the same region: positive (black) and negative (gray) anomaly. The contour interval is 5 cm from −35 to 35 cm.

processing can be found in Le Traon et al. (1998) and Le Traon and Ogor (1998). The geoid model described in Lemoine et al. (1997) is subtracted to estimate the height relative to the geoid. This is the same dataset used in Marshall et al. (2006).

The geostrophic relation is used to convert the sea surface height h to a geostrophic velocity, $(u_g, v_g) = f^{-1}g(-\partial_y h, \ \partial_x h)$. Because of variations in f and the presence of boundaries where the total normal velocity is set to zero ($\mathbf{u} \cdot \mathbf{n} = 0$, where $\mathbf{n}$ is a unit vector normal to the boundary), the geostrophic relation will yield a velocity field that is divergent. This is a serious practical problem, because Nakamura's approach, used later to estimate eddy diffusivities, can be applied only to divergenceless velocity fields. We therefore set $\mathbf{u} = \mathbf{u}_g + \nabla\chi$, where $\nabla\chi$ is a (divergent) adjustment to the altimetric velocity that renders $\mathbf{u}$ nondivergent and zero across

meridional boundaries and periodic across zonal boundaries. The method is borrowed from Marshall et al. (2006) and results in a very small modification of the velocity field.

We focus our study on two regions in the Southern Ocean. First, we consider a patch in the Pacific sector of the Southern Ocean between 66° and 30°S, 125° and 150°W, shown in Fig. 1 (note that Fig. 1 shows the sea surface height mean and anomaly as measured by the altimeter before any correction is applied to make the flow divergenceless and consistent with the zonal and meridional boundary conditions). The patch is chosen to be away from strong western boundary currents and to be characterized by a broad ACC, a configuration consistent with the broad rectilinear current model used to derive the expressions for $K_\perp$. The zonal mean and eddy kinetic energies in the patch are shown in Fig. 2. The
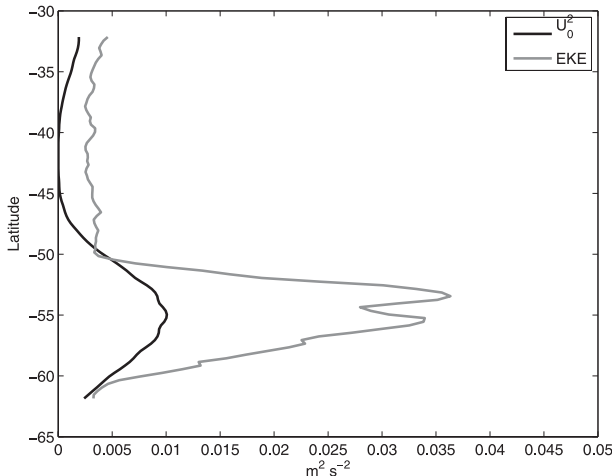
FIG. 2. Zonally averaged mean flow squared (black) and EKE (gray) at the surface for a patch in the Pacific sector of the Southern Ocean between 66° and 30°N and between 125° and 150°W.

mean flow in the patch is predominantly zonal, directed eastward flowing at ~10 cm s^{-1} at 55°S in the ACC and at ~4 cm s^{-1} at 30°S in the subtropical gyre latitude band. EKE is an order of magnitude greater in the ACC region than in the region north of it.

The second region that we consider is the full Southern Ocean between 30° and 65°S. Both mean and anomaly of sea surface height are shown in Fig. 3. The streamline averages of mean and eddy kinetic energies are plotted in Fig. 4. The ACC flow pattern is characterized by alternating broad jets, where our theory applies, and sharp meandering jets, where our theory is not formally valid. Despite these formal limitations, the expression for $K_\perp$ remains qualitatively and quantitatively accurate when applied to the whole Southern Ocean (see below).

As a first step, we use the altimetric data to support our ansatz that the eddy interaction time scale γ^{-1} is proportional to the eddy strain rate as written in (15). The calculation is performed for the Pacific sector patch. Following Garrett (1983), the rms strain rate is defined as $\langle (\partial_x u)^2 + (\partial_y v)^2 \rangle^{1/2}$; the average $\langle \cdots \rangle$ is taken over a full year and the longitudinal extent of the patch, whereas derivatives are computed by finite differentiation of the altimetric velocity field. The rms strain rate conveniently reduces to $(2\kappa^2 \, \text{EKE})^{1/2}$ for a monochromatic wave field. The eddy interaction time scale is inferred from the Eulerian autocorrelation function for the meridional velocity (we are interested in meridional eddy transport). In Fig. 5, we show the autocorrelation function for all latitudes in the Pacific sector patch. The autocorrelation drops to zero after 20 days in the core of the ACC, but only after 50 days north of it. The same range of variations is observed in rms strain rate: the thick line in Fig. 5

shows the inverse rms strain rate multiplied by 5. The eddy decorrelation time scale is expected to be proportional to the time at which the Eulerian autocorrelation drops to zero; hence, Fig. 5 supports the relationship in (15).

4. Estimates of surface eddy diffusivity in the Southern Ocean

The goal of this section is to test the skill of the expression for the eddy diffusivity in (19) against data. We take the following approach. We diagnose $K_\perp$ from altimetric observations using the diagnostic approach of Nakamura. The diagnostic estimates of $K_\perp$ are then compared with the theoretical prediction of (19). The comparison is repeated for the Pacific sector patch and for the whole Southern Ocean.

a. Estimates of eddy diffusivity using Nakamura's diagnostic approach

Nakamura (1996) showed that it is possible to estimate eddy diffusivities by numerically advecting a passive tracer with an observed velocity field. The approach has been used extensively in atmospheric studies (e.g., Haynes and Shuckburgh 2000a,b) and more recently in oceanographic studies (e.g., Marshall et al. 2006). Here, the velocity field is estimated from the altimetric measurement of sea surface height. The geostrophic velocity field is made nondivergent as described above, a requirement for Nakamura's approach to work. The basic idea is that a nondivergent velocity field is area preserving. Therefore, only molecular (or numerical) diffusion can change the area enclosed by tracer contours. Geostrophic eddies, however, increase the molecular fluxes by twisting and folding tracer contours so that the interface available for molecular diffusion is enhanced. Nakamura showed that, in two dimensions, the eddy enhancement over the background molecular diffusion is given by

$$K_\perp = \mu L_{\text{contour}}^2 / L_0^2,$$

where μ is the small background constant diffusivity used in the numerical code, L_{contour} is the observed length of a tracer contour, and L_0 is the minimum (unstrained) length of the contour. The strategy is then to numerically advect an idealized tracer using the velocity field derived from the altimeter and estimate L_{contour} from the resulting tracer distribution. The approach does not depend on details of the numerical dissipation because $K_\perp$ becomes independent of μ for μ sufficiently small. Marshall et al. (2006) used this technique to estimate $K_\perp$ across the ACC in the Southern Ocean. The technique cannot be used to estimate the eddy diffusivity along mean currents because tracers are quickly homogenized along mean streamlines.
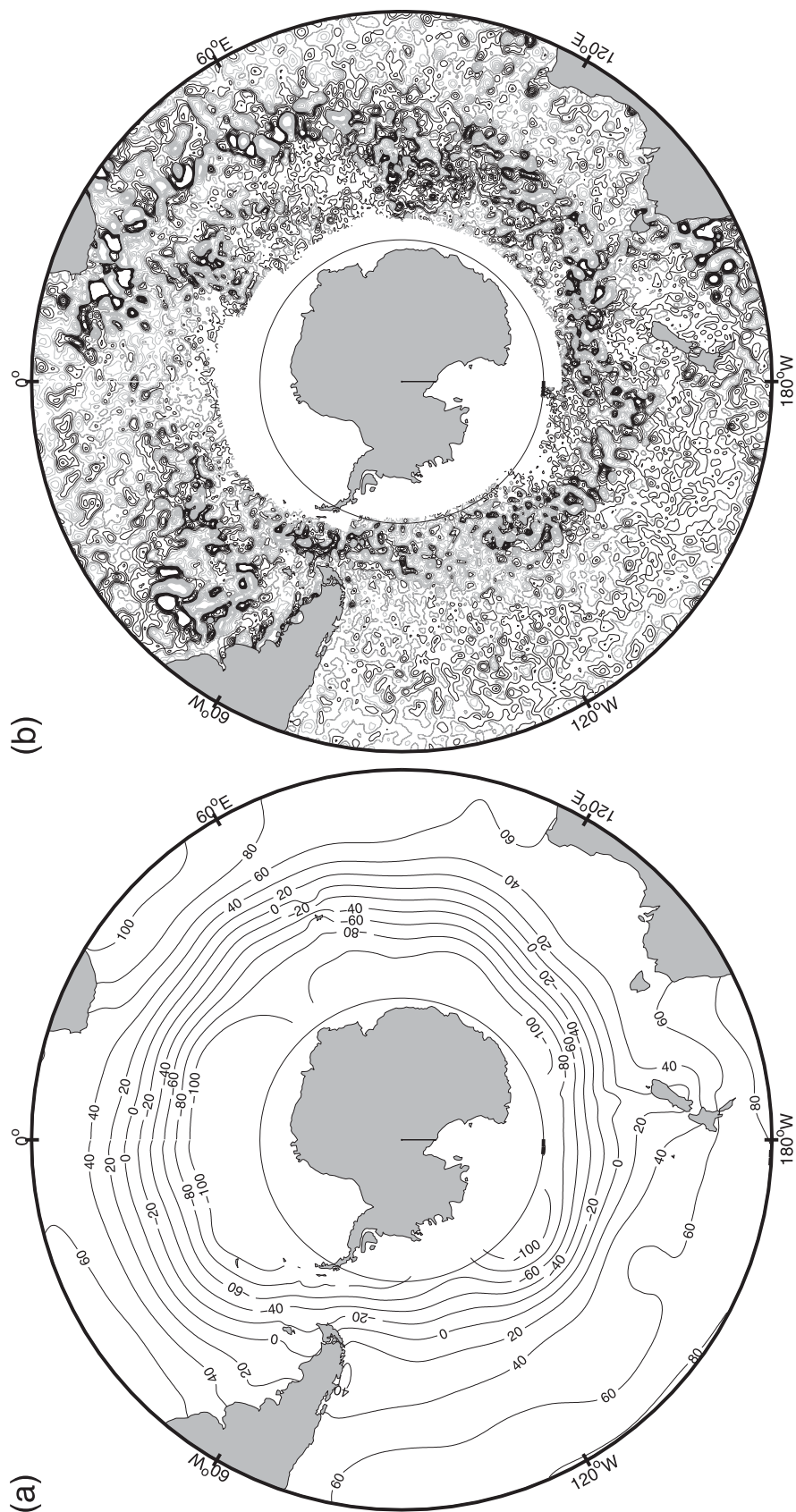
FIG. 3. (a) Mean sea surface height of the Southern Ocean in centimeters. The contour interval is 20 cm from −100 to 100 cm. (b) Snapshot of sea surface height anomaly of the Southern Ocean: positive (black) and negative (gray) anomaly. The contour interval is 5 cm from −35 to 35 cm.
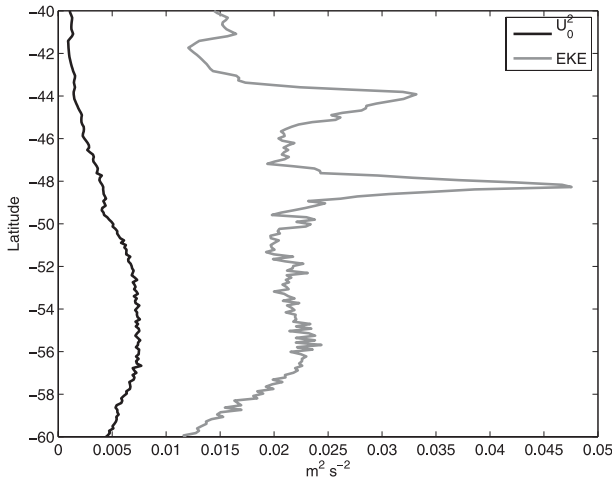
FIG. 4. Mean surface velocity squared (black) and EKE (gray) averaged along mean streamlines for the whole Southern Ocean.

First, we apply Nakamura's approach to the Pacific sector patch. To resolve fine filaments in the tracer field, the geostrophic velocities are interpolated onto a higher resolution grid of $1/20°$. The Massachusetts Institute of Technology general circulation model (MITgcm) (Marshall et al. 1997) is used to advect a passive tracer c by the prescribed nondivergent velocity field $\mathbf{u}$, solving the following tracer advection–diffusion equation:

$$c_t + \mathbf{u} \cdot \nabla c = \mu \nabla^2 c,$$

where μ is a background numerical diffusivity. The computational domain is zonally periodic with no-flux boundary conditions imposed at the southern and northern boundaries. A linear meridional tracer distribution with no variation in zonal direction is chosen as an initial condition. The value of the background numerical diffusivity is set to $\mu = 10$ m^2 s^{-1}.

We make two different tracer advection–diffusion calculations: one using the full velocity field, which includes the mean flow and the eddy fluctuations, and the other using only the eddy fluctuations. The mean flow is computed from the sum of the mean geoid and the 1-yr time-mean sea surface height from the altimetry using the geostrophic relation. (In the Pacific sector calculations described below, the definition of mean also includes a zonal average.) The eddy fluctuations are departures from the mean flow. Eddy diffusivities estimated with advection by the full velocity field will be referred to as $K_\perp$, while eddy diffusivities estimated without advection by the mean flow will be referred to as $K_{U=0}$.

The altimeter data yield one set of flow fields for each 10-day period; the velocity fields in between the 10-day intervals are obtained through linear interpolation. We
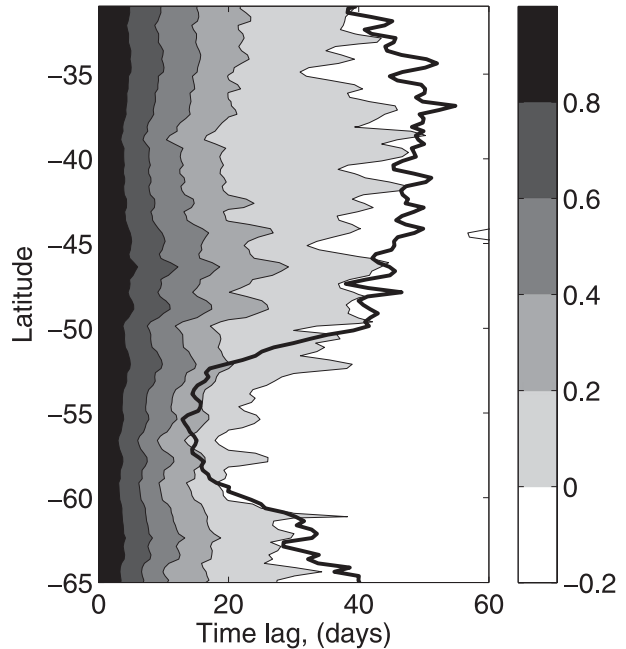

FIG. 5. Contours of the autocorrelation function for the meridional velocity as measured by the altimeter in the Pacific sector patch shown in Fig. 1. The autocorrelation function is computed at every location and averaged along longitude sections for each latitude. The thick black line shows the inverse rms strain rate averaged along each longitude section multiplied by a factor of 5.

calculate a set of 36 flow fields covering one annual cycle (from October 1996 to October 1997). The tracer is advected for a total of 2 yr, repeating the 1-yr cycle twice. After an initial transient period of a few months, Nakamura's estimate of the eddy diffusivity equilibrates. Tracer fields from the last year of calculations are used to diagnose $K_\perp$. There are some seasonal fluctuations of $K_\perp$ throughout the year caused by variability of the velocity fields, but they are weak. Here we present only results for annual averages.

Estimates of eddy diffusivity for the Pacific sector patch are shown in Fig. 6 as a function of "equivalent latitude." The equivalent latitude is approximately the mean latitude of a tracer contour, and the mean tracer contours are very close to mean streamlines (see Nakamura 1996). There are significant latitudinal variations in eddy diffusivity. In the calculation without advection by the mean flow, $K_{U=0}$ peaks at 4500 m^2 s^{-1} in the core of the ACC (50°–60°S) and drops to around 1000 m^2 s^{-1} to the north. This is consistent with the Holloway (1986) prediction that the eddy diffusivity is largest where the EKE is largest—that is, in the core of the ACC. The picture changes substantially when advection by the mean flow is added to the calculation. In the core of the ACC, $K_\perp$ is strongly suppressed, by up to a factor of 2–3, while it remains nearly unchanged around 40°S where

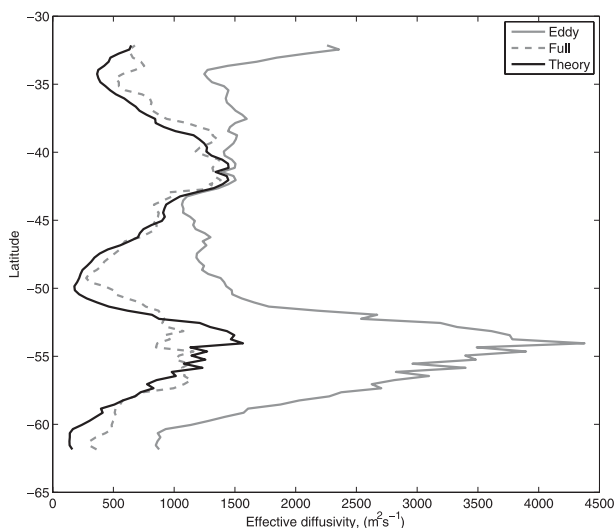FIG. 6. Meridional eddy diffusivity ($m^2\,s^{-1}$) from calculations for a patch in the Pacific Ocean between 125° and 150°W: eddy diffusivity estimated from Nakamura's approach using only geostrophic eddy velocity (solid gray) and using full geostrophic velocity (dashed gray); theoretical prediction in (20) (solid black).
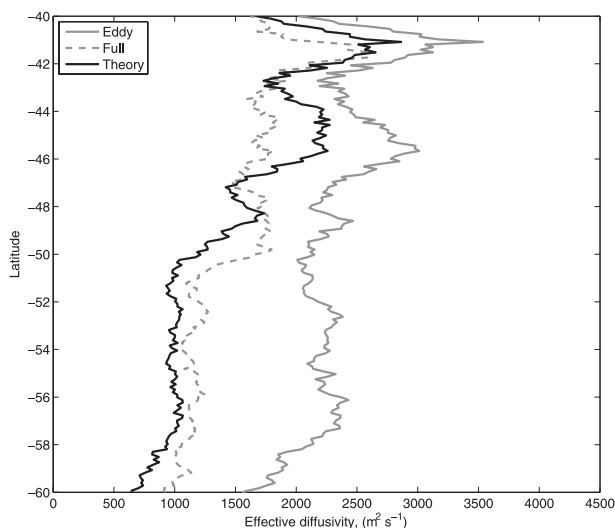

FIG. 7. Eddy diffusivity ($m^2\,s^{-1}$) across mean streamlines from calculations for the full Southern Ocean: eddy diffusivity estimated from Nakamura's approach using only the geostrophic eddy velocity (solid gray) and using the full geostrophic velocity (dashed gray); theoretical prediction in (20) (solid black).

mean currents are absent. Some suppression is also observed north of 40°S as a result of the mean flows associated with the subtropical gyres. The suppression in the ACC is so strong that the maxima in $K_\perp$ are no longer collocated with maxima in EKE.

Contrary to what might be expected from linear wave theory, we do not observe $K_\perp$ enhancement in correspondence of critical layers. Critical layers are located on the flanks of the ACC, at latitudes where $c_w = U_0$, in the simulations with advection by the full velocity field, and they move closer to the core of the ACC, at latitudes where $c_w = 0$, in the simulations that do not include the mean flow. Figure 6 shows that there is no latitude where $K_\perp$ is enhanced over $K_{U=0}$, not even on the flanks of the ACC where the critical layers should have drifted by including advection by the mean flow.

Next, we apply the Nakamura approach to the whole Southern Ocean. The basic setup of the tracer advection–diffusion calculation is similar to the one used for the patch in the Pacific sector. Owing to the increased domain size, we use a coarser grid resolution of ¼° and a higher background molecular diffusivity $\mu = 100\ m^2\,s^{-1}$. Marshall et al. (2006) show that the estimate of eddy diffusivity is not very sensitive to grid resolution and μ for the range of values used here. The mean sea surface height shown in Fig. 3 is chosen as the initial tracer distribution.

Estimates of $K_\perp$ from the full Southern Ocean are shown in Fig. 7. In the calculation without advection by the mean flow, $K_{U=0}$ is much more uniform with latitude

than the corresponding estimate for the Pacific sector patch. It varies from about 2000 $m^2\,s^{-1}$ south of 50°S, in the ACC region, to larger values, up to 3000 $m^2\,s^{-1}$, north of it. It is important to note that the eddy diffusivity diagnosed using Nakamura's approach is an average value along a tracer contour. The large values of $K_{U=0}$ in the ACC region are generated by the strong eddy field throughout the ACC, whereas the even larger values north of the ACC are due to the vigorous eddy field associated with the boundary currents of the Southern Hemisphere subtropical gyres.

The eddy diffusivity is strongly suppressed when the mean flow is included into the calculation for the full Southern Ocean (Fig. 7): $K_\perp$ is reduced to about 1000 $m^2\,s^{-1}$ in the ACC region and to 1500–2000 $m^2\,s^{-1}$ north of it. The largest $K_\perp$ is still found north of the ACC rather than in its core. The magnitude and variations of $K_\perp$ are very similar to those reported by Marshall et al. (2006), who applied the same technique to study mixing in the Southern Ocean. Once again, there is no clear signature of enhancement at critical layers; that is, regions where $K_\perp$ is enhanced over $K_{U=0}$.

b. Estimates of eddy diffusivity using a new prognostic approach

The expression for the eddy diffusivity that we derived in section 2 suggests that the presence of a mean current suppresses $K_\perp$ compared to the value it would have in the absence of a mean current. The expression (19) predicts that the estimates of eddy diffusivity with ($K_\perp$) and

without ($K|_{U=0}$) advection by the mean flow should be related according to

$$K_\perp = \frac{K|_{U=0}}{1 + 2d_2 |\overline{\nabla h}|^2 / \overline{|\nabla h'|^2}}. \tag{20}$$

The continuous black line in Fig. 6 shows $K_\perp$ as estimated from Eq. (20) for the Pacific sector patch: $K|_{U=0}$ is taken from Nakamura's calculation for the simulation without advection by the mean flow and $\overline{h}$ and h' are computed from altimetric data. The resulting map of $K_\perp$ values is then averaged along mean streamlines. The continuous black line matches the gray dashed line, which is the estimate of $K_\perp$ taken from Nakamura's calculation for the simulation with advection by the mean flow, remarkably well. The model (20) captures well the modulation of $K_\perp$ by the mean flow at all latitudes for a constant value of the parameter $d_2 = 4$. We conclude that the suppression of the diffusivity in the Pacific sector patch is indeed related to the decrease in mixing length associated with advection by the mean ACC jets.

Figure 7 shows the same comparison for the whole Southern Ocean. All eddy diffusivity estimates are averaged along mean streamlines. The suppression is again captured remarkably well for the same value of $d_2 = 4$. This further builds our confidence that the simple theoretical model derived in section 2 captures qualitatively and quantitatively the suppression of $K_\perp$ by the mean jets that constitute the ACC. The spikes in $K_\perp$ north of the ACC latitude band are associated with high mixing rates in the boundary currents of the Southern Hemisphere subtropical gyres. The spikes do not appear in the estimates based on Nakamura's approach because of the different averaging. Nakamura's $K_\perp$ is the ratio of the along-stream-averaged tracer flux divided by the along-stream-averaged tracer gradient (the ratio of the averages), whereas the $K_\perp$ estimated from Eq. (20) is the along-stream average of the local ratio of tracer flux and tracer gradient (the average of the ratios). The average of the ratios is more sensitive to high mixing values than the ratio of the averages.

The careful reader should be puzzled by one aspect of our interpretation of the results so far. Theory predicts that the suppression of $K_\perp$ in the jet is proportional to the ratio of the squared intrinsic eddy phase speed and the eddy kinetic energy, $(c_w - U_0)^2/\text{EKE}$. Eddies propagate at the speed c_w both in the numerical simulations using the full velocity field and in those using only the eddy velocity field. Some suppression of $K_\perp$ should therefore occur in both simulations. However, Smith and Marshall (2009) show that typically in the Southern Ocean the eddy phase speed is small; that is, $c_w \ll U_0 \ll \text{EKE}^{1/2}$.
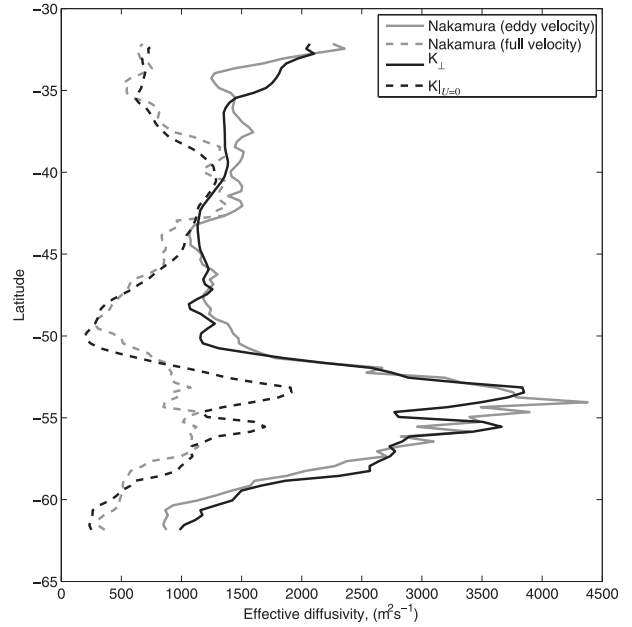

FIG. 8. Meridional eddy diffusivity ($\text{m}^2 \, \text{s}^{-1}$) for a path in the Pacific sector of the Southern Ocean between 125° and 150°W estimated directly from altimetric observations: eddy diffusivity estimated from Nakamura's approach using only the geostrophic eddy velocity (solid gray) and using the full geostrophic velocity (dashed gray); theoretical prediction in (19) in the absence of mean flow (solid black) and with the mean flow (dashed black).

Suppression of mixing is therefore substantial only in the simulations with advection by the mean flow where $(c_w - U_0)^2/\text{EKE} \approx U_0^2/\text{EKE}$. In the simulations without advection by the mean flow the suppression of mixing is much weaker, being proportional to $c_w^2/\text{EKE} \ll U_0^2/\text{EKE}$.

Next, we test our expression for the eddy diffusivity based solely on altimetric data [Eq. (19)] against the Nakamura calculations without advection by the mean flow. This is essentially a test of the Holloway (1986) model, which is expected to have skill in the absence of mean flows. The solid black lines in Figs. 8 and 9 show that (19) well reproduces the tracer-based estimates for simulations with no advection by the mean flow, that is, $K|_{U=0}$, both in the Pacific sector and in the global Southern Ocean calculations. This supports our claim that Holloway's model holds in the absence of mean flows, when the mixing length ℓ is indeed proportional to the eddy scale. However, Holloway's model fails when mean flows are present because ℓ is reduced. The reduction of ℓ and $K_\perp$ by mean currents is an important correction, because large h' fluctuations in sea surface height tend to occur in regions where mean flows are large: the turbulent fluctuations result from instabilities of the mean currents. Hence, Holloway's scaling leads to a consistent and large overestimate of $K_\perp$.
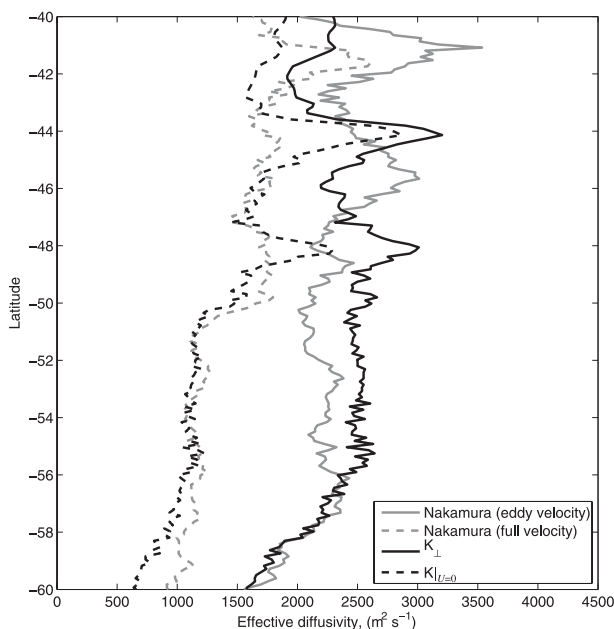
FIG. 9. Eddy diffusivity ($m^2 s^{-1}$) across mean streamlines for the full Southern Ocean estimated directly from altimetric observations: eddy diffusivity estimated from Nakamura's approach using only the geostrophic eddy velocity (solid gray) and using the full geostrophic velocity (dashed gray); theoretical prediction in (19) in the absence of mean flow (solid black) and with the mean flow (dashed black).

In Figs. 8 and 9, we compare the prediction of the altimetric-based estimate for $K_\perp$, as given by (19), against the estimate based on Nakamura's approach for the simulations using advection by the full velocity field (mean plus eddies). We have already shown that expression (19) captures the suppression of $K_\perp$ by the mean currents and reproduces Holloway's estimates for mixing in the absence of mean currents. It should therefore come as no surprise that our expression (19) does a remarkable job at reproducing the estimates based on Nakamura's approach. A good matching is achieved at all latitudes for both the Pacific sector patch and the full Southern Ocean with $d_1 = 0.32$ and $d_2 = 4$. The upshot of this work, therefore, is that variability of $K_\perp$ at the ocean surface can be directly inferred from altimetry according to the expression in (19).

The value of d_1 is comparable to those found in previous work relating $K|_{U=0}$ to sea surface height variability. Holloway (1986) found that $d_1 = 0.4$ was a good predictor of the eddy diffusivities. Kushner and Held (1998) tested the approach with atmospheric data and found that it captured the spatial variation of the diffusivity with optimum values of d_1 varying between 0.2 and 0.4. Karsten and Marshall (2002) also tested the method using numerical simulations and altimetric measurements and found that $d_1 = 0.26$ gave the best fit.

5. Conclusions

The main result of this paper is that the surface eddy diffusivity across the Antarctic Circumpolar Current in the Southern Ocean can be computed from sea surface height data using the expression

$$K_\perp = 0.32 \frac{g}{|f|} \frac{(\overline{h'^2})^{1/2}}{1 + 8|\nabla \overline{h}|^2 / \overline{|\nabla h'|^2}}, \qquad (21)$$

where h is the sea surface height, the overbar denotes a temporal average at a fixed point, and primes departures from that average. The expression reduces to the one derived by Holloway (1986) in the absence of mean flows; that is, $\nabla \overline{h} = 0$. The new result is that that eddy mixing across a permanent current is not simply proportional to eddy fluctuations in sea surface height, but it is suppressed when the ratio of mean kinetic energy to eddy kinetic energy is sufficiently large; that is, when $8|\nabla \overline{h}|^2 / \overline{|\nabla h'|^2}$ is $O(1)$. Although suppression of eddy mixing by mean currents has been noted before (e.g., Bower 1991; Marshall et al. 2006), we are not aware of studies that quantify the degree of suppression. Indeed, suppression of eddy mixing by mean currents is presently ignored in parameterizations of eddy diffusivities, and it is likely a major limitation in the skill of numerical models used for climate studies.

We validated the expression in (21) against estimates of cross-current $K_\perp$ based on the Nakamura approach. Our analysis focused on the Southern Ocean where cross-current eddy mixing plays a crucial role in driving the meridional overturning circulation (e.g., Marshall and Radko 2003) and the meridional heat transport (e.g., Keffer and Holloway 1988). However, our results should apply to eddy mixing across other permanent currents such as the western boundary currents and equatorial currents.

In Fig. 10, we show two-dimensional maps of eddy diffusivity based on our expressions applied to altimetric observations from the Southern Ocean. The top panel shows $K_\perp$ as given by the full expression in (21). The middle panel shows K_0; that is, the eddy diffusivity given by Holloway's expression, which does not include suppression by the mean currents. Holloway's expression is recovered by setting the denominator in (21) to one. The third panel shows the ratio of the first two panels: that is, the degree of suppression of mixing by mean currents. Holloway's expression gives the largest values of mixing in the ACC and in the western boundary currents of the subtropical gyres. The picture changes dramatically when we include the effect of mean currents on eddy mixing. Suppression by the ACC currents is so strong that $K_\perp$ is rarely larger in the ACC than north of it.
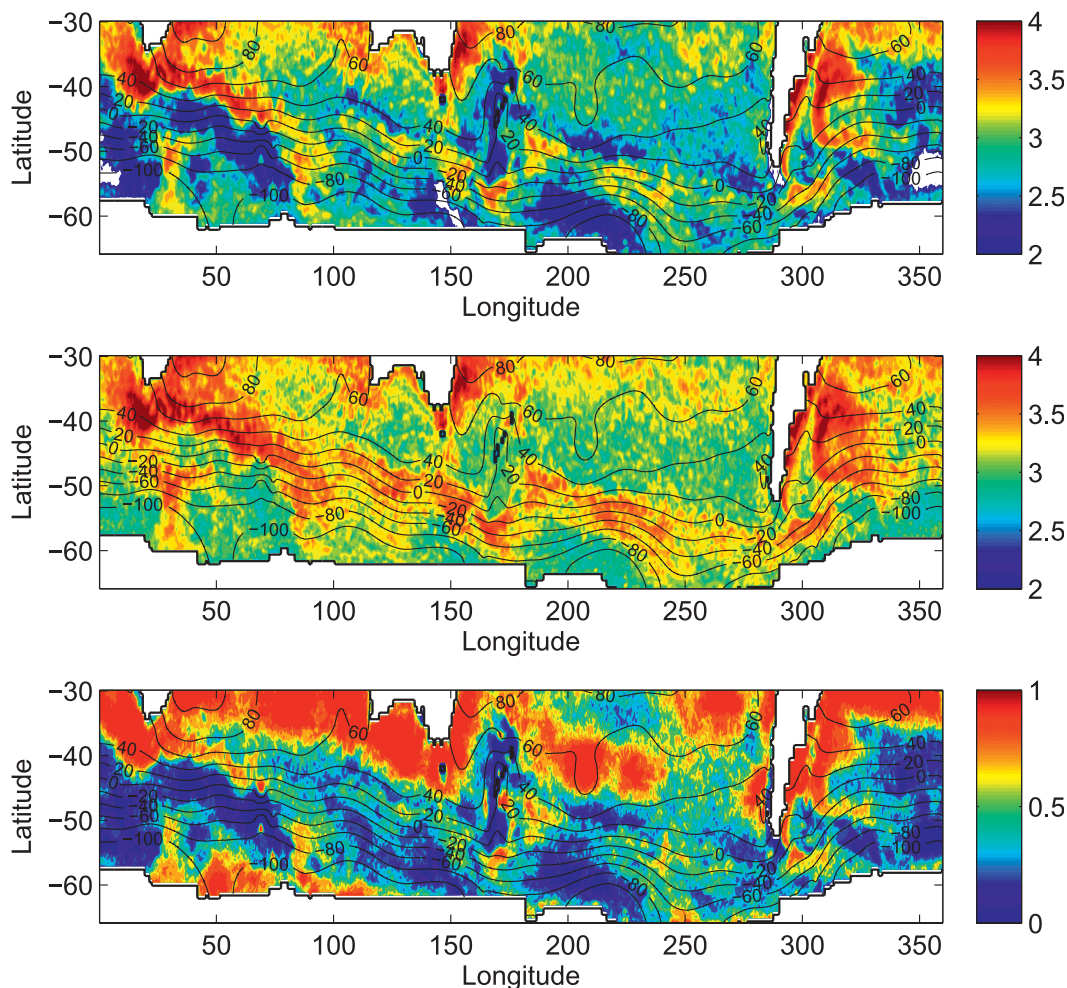
FIG. 10. Maps of the eddy diffusivity [$\log_{10}$ (m^2 s^{-1})] for the full Southern Ocean estimated directly from altimetric observations: logarithm of the eddy diffusivity estimated (top) from both anomaly and time-mean sea surface height through (19) and (middle) from the sea surface height anomaly ignoring suppression by the mean flow; (bottom) suppression of the eddy diffusivity by the mean flow equal to the ratio of the diffusivities shown in the middle and top panels.

Variations in cross-current eddy mixing have been recently reported by Marshall et al. (2006) in a diagnostic study of mixing in the Southern Ocean. Marshall et al. speculated that the increase in $K_\perp$ on the equatorward flank of the ACC was associated with the development of critical layers at the edge of the ACC. Linear wave theory does, indeed, suggest that eddy mixing is enhanced at critical layers where the phase velocity of the waves matches the mean current speed, typically at the edge of strong currents. Here, we moved beyond a diagnostic study and developed a dynamical model to interpret the variations in $K_\perp$. Our work shows that the increase in eddy mixing north of the ACC is not the result of a critical layer enhancement on the flank of the ACC jets, but it is rather the result of the strong suppression of $K_\perp$ in the core of the ACC. Naveira Garabato et al. (2010, unpublished manuscript) confirm our results through

analysis of hydrographic and altimetric data. They also find that the eddy diffusivity is suppressed in the core of ACC jets, whereas there is no signature of critical layer enhancement on the flanks of the jets.

There is extensive literature on the role of critical layers in driving eddy mixing across mean currents in geophysical flows (Pierrehumbert 1990; Randel and Held 1991; Smith and Marshall 2009). This literature relies on the observational evidence that geostrophic eddies often propagate at a different speed than the mean currents in which they are imbedded. One can show that, if eddies are linear, that is, they are waves, no mixing can occur in most of the domain because wave stirring is completely reversible. Mixing can only occur in critical layers at the edge of currents where the eddies/waves drift at the same speed as the mean current. This results in a permanent stretching of tracer filaments that is eventually

arrested by molecular mixing when the tracer filaments become infinitesimally thin. Extensions of these ideas based on dynamical system theory, the "chaotic mixing" literature, generalizes the kinematic framework. Most notably, the linear concept of critical layers is replaced by extended stochastic layers. We showed that, when dynamics is brought into the picture, the role of critical/stochastic layers in modulating mixing is not as central to the mixing problem. Nonlinear interactions continuously exchange energy among different waves; therefore, waves have a finite lifetime. Eddies with a finite lifetime are not fully reversible and are efficient mixers. In this scenario, mixing occurs everywhere, but mixing is suppressed in regions where the mean flow is sufficiently fast to advect tracer out of the eddy faster than the eddy lifetime and hence reduce the time for which eddies can stir the tracer. Critical/stochastic layers are, instead, regions where suppression by the mean current does not occur.

The main result of this paper is that eddy mixing is suppressed in the core of jets because eddies propagate at a speed proportional to, but smaller than, that of the mean flow in which they are embedded. The simple quasigeostrophic model used in this paper to derive an expression for $K_\perp$ provides a dynamical framework to understand the relationship between mean currents and eddy phase speeds. Geostrophic eddies are typically generated through baroclinic instabilities of strong jets such as the oceanic western boundary currents, the Southern Ocean Antarctic Circumpolar Current, and the atmospheric jet stream. These eddies propagate downstream of the mean jet at a speed substantially smaller than the jet itself: this is a fundamental result of baroclinic instability theory (Pedlosky 1987) and it is supported by observations (Smith and Marshall 2009). The relationship between eddy phase speed and mean jet speed is at the core of the expression in (21). The suppression in the denominator of expression (21) for $K_\perp$ is proportional to the ratio of the squared intrinsic phase speed of the eddies and the eddy kinetic energy. Chelton et al. (2007) show that this ratio also characterizes the degree of nonlinearity of the eddies; hence, suppression is expected only if eddies are nonlinear. It is because of the dynamical relationship between the eddy phase speed and the mean current speed that the suppression in $K_\perp$ can be expressed as the ratio of the mean and eddy kinetic energies. This relationship would be lost in a purely kinematic framework that arbitrarily prescribes the differences between eddy and mean current speeds.

Our results are consistent with the idea that mixing is suppressed in regions with strong potential vorticity gradients (e.g., Dritschel and McIntyre 2008). We have shown that suppression of mixing is associated with the propagation of eddies along mean currents. The eddies propagate along the mean currents because they follow the potential vorticity gradients associated with the mean currents. Hence, suppression of mixing does indeed require a potential vorticity gradient. Our results, instead, depart from previous literature that finds suppression of mixing where the horizontal shear in the mean currents is large (e.g., d'Ovidio et al. 2009). Our analytical model ignores lateral shears in the mean current, because the assumption of a scale separation between mean and eddy scales implies that any mean flow shear is weak. Nakamura's diagnostics support our assumptions because the suppression of mixing is strongest in the core of the ACC, where the mean shear is weakest, and not on its flanks, where the mean shear is largest.

Our final expression for eddy diffusivity $K_\perp$ at the ocean surface does not include enhancement at lateral critical layers. However, the more general expression for $K_\perp$ in (14) allows for local maxima at critical layers if eddies radiate laterally to regions where their phase speed matches the mean current speed. Critical layers were eliminated in our theory by assuming that eddies do not drift too far from their generation region. The success of the final expression for $K_\perp$ at reproducing the observed patterns of eddy mixing seems to vindicate the assumption. Nevertheless, the role of critical layers could be more thoroughly explored by using published estimates of eddy phase speeds with our more general expression for $K_\perp$ in (14).

Although no evidence was found for large values of $K_\perp$ at critical layers in the horizontal, our theory still allows for local maxima in $K_\perp$ at critical layers in the vertical. Ocean eddies are known to have deep vertical structure (Wunsch 1997), whereas the speed of the mean ACC jets decays rapidly through the upper kilometer of the ocean. Hence, the eddy phase speed will match the mean current speed at some depth in the vertical, as shown by Smith and Marshall (2009). This depth corresponds to a critical layer in the vertical where suppression of mixing by the mean current ceases and $K_\perp$ has a local maximum. Abernathey et al. (2010) have recently diagnosed the three-dimensional structure of $K_\perp$ in the Southern Ocean by applying Nakamura's technique to an ocean reanalysis. They find enhancement of mixing at depth in correspondence with critical levels where the eddy phase speed matches the mean jet speed. Bower and Lozier (1994) find similar results in analysis of float data in the Gulf Stream region: mixing in the core of the jet is high at depth, while it is suppressed at the surface. Our simple model is consistent with these results, as shown in expression (14).

Finally, we have shown that the expression in (21) is accurate in a region where the ACC is broad and rectilinear, and it also captures the overall mixing rate across the whole ACC. We expect our formula to break locally in

regions where the mean currents meander strongly due to barotropic instabilities or topographic steering. In these regions both the assumption of scale separation between the mean and eddies and the relationship between mean current speed and eddy phase speed break. MacCready and Rhines (2001) argue for strong mixing in the wake of major topographic features based on idealized models. Their result is confirmed by the observational study of A. C. Naveira Garabato et al. (2010, unpublished manuscript), who find that suppression of mixing by the mean ACC jets is observed only away from topographic features. These regions might be of global importance because they represent the few leaks through the mostly impermeable ACC. We intend to study these regional variations of $K_\perp$ in future work.

APPENDIX

A Solution of the Stochastic Surface Quasigeostrophic Model

The stochastic surface QG model used in this paper is based on the equations:

$$\partial_t b + U_0 \partial_x b - \Gamma \partial_x \psi = \mathcal{U} N \sqrt{\gamma}\, \mathrm{Re}[r(t) e^{i(kx+ly)}] \quad - \gamma b, \quad z = 0 \tag{A1}$$

and

$$\partial_x^2 \psi + \partial_y^2 \psi + \frac{f^2}{N^2} \partial_z^2 \psi = 0, \quad z < 0. \tag{A2}$$

Solutions for the streamfunction ψ and the buoyancy b that satisfy (A1) and (A2) are written as

$$\psi(x, y, z, t) = \frac{\mathcal{U}}{\kappa} \mathrm{Re}\left\{a(t) \exp\left[i(kx+ly) + \frac{N\kappa}{f} z\right]\right\},$$
$$b(x, y, t) = N\mathcal{U}\, \mathrm{Re}\left\{a(t) \exp\left[i(kx+ly) + \frac{N\kappa}{f} z\right]\right\}. \tag{A3}$$

Substituting these expressions in the equation for buoyancy in (A1) gives the evolution equation for the complex function $a(t)$,

$$\frac{da}{dt} + \left[\gamma + ik\left(U_0 - \frac{\Gamma}{N\kappa}\right)\right] a = \sqrt{\gamma}\, r(t). \tag{A4}$$

Integrating in time from $-\infty$ to t (the time integration is started at $-\infty$ to eliminate dependence on spurious initial transients), one obtains

$$a(t) = \sqrt{\gamma} \int_0^\infty r(t-\tau) e^{-(\gamma+ikc_w)\tau}\, d\tau, \tag{A5}$$

where c_w is the wave speed of a surface edge wave given in (8).

The solution for the tracer concentration c is obtained by assuming that the tracer has the same spatial structure as the streamfunction in (A3) but a different time-dependent amplitude. The solution is then inserted into (10). The nonlinear term $J(\psi, c)$ vanishes, and the tracer equation can be solved by the same method used for (A4). The solution has the form

$$c(x, y, z, t) = -\Gamma_c \frac{k}{\kappa} \mathcal{U}\, \mathrm{Re}\Bigg(\int_0^\infty ia(t-\tau)$$
$$\times \exp\left\{i[kx + ly - kU(z)\tau] + \frac{N\kappa}{f} z\right\} d\tau\Bigg), \tag{A6}$$

with $a(t)$ given in (A5).

We can now use the solutions for c and $v = \partial_x \psi$ to compute the expected value of $\langle vc \rangle$. First, let us write $c = (C + C^*)/2$ and $v = (V + V^*)/2$, where C and V are the full complex solutions for c and v, that is, the arguments inside the real operator in (A3) and (A6), and the asterisk denotes complex conjugates. The expected value of vc is given by

$$\langle vc \rangle = \frac{1}{4}\langle VC^* + V^*C \rangle. \tag{A7}$$

The terms $\langle VC \rangle$ and $\langle V^*C^* \rangle$ vanish because they include the expected values of $\langle r(t)r(t') \rangle$ and its complex conjugate, which are zero for a white noise stochastic process. Substituting the expressions for V and C, we have

$$\langle V^*C \rangle = -\Gamma_c \frac{k^2}{\kappa^2} \mathcal{U}^2 \gamma \exp\left(2\frac{N\kappa}{f} z\right) \int_0^\infty dt_1 \int_0^\infty dt_2 \int_0^\infty d\tau e^{-\gamma(t_1+t_2)-ikc_w(t_2-t_1)-ikU(z)\tau} \langle r^*(t-t_1)r(t-\tau-t_2) \rangle$$
$$= -\frac{1}{2}\Gamma_c \frac{k^2}{\kappa^2} \mathcal{U}^2 \frac{1}{\gamma - ik[c_w - U(z)]} \exp\left(2\frac{N\kappa}{f} z\right), \tag{A8}$$

where we used the condition that $\langle r^*(t-t_1)r(t-\tau-t_2)\rangle = \delta(\tau+t_2-t_1)$ for $t_1 > t_2$ and $\langle r^*(t-t_1)r(t-\tau-t_2)\rangle = 0$ for $t_1 < t_2$. This solution can then be used to compute the expected value of $\langle vc \rangle$,

$$\langle vc \rangle = -\frac{1}{4}\mathcal{U}^2\frac{k^2}{\kappa^2}\frac{\gamma}{\gamma^2+k^2[c_w-U(z)]^2}\ \exp\!\left(2\frac{N\kappa}{f}z\right)\Gamma_c. \tag{A9}$$

This expression is the starting point for the discussion of the eddy diffusivity in this paper.

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